

**Problem Set No. 4**

**Out:** Wednesday, September 30, 2009

The homework problems are for practice only. Solutions are posted in a separate file. Please work on the problems and be prepared to ask questions related to this homework in the recitation of Tuesday, October 6, 2009 (4:00–5:30pm in Room 1-379).

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**Problem 1**

The moments and products of inertia with respect to a given rectangular coordinate system of a certain body are

$$\begin{aligned} I_x &= 9 & I_{xy} &= 4 & I_{xz} &= 0 \\ I_y &= 3 & I_{yz} &= 0 \\ I_z &= 5 \end{aligned}$$

Find the principal moments of inertia and locate the principal axes for the origin of the given coordinate system.

**Problem 2**

A billiard ball, initially at rest, is given a sharp impulse by a cue. The cue is held horizontally a distance  $h$  above the centerline. The ball leaves the cue with a speed  $v_0$  and eventually acquires a final speed of  $\frac{9}{7}v_0$ . Show that  $h = \frac{4}{5}R$ , where  $R$  is the radius of the ball.

**Problem 3**

A thin rectangular plate of mass  $M$ , sides  $a$  and  $b$ , rotates about an axis along its diagonal with angular velocity  $\omega$ .

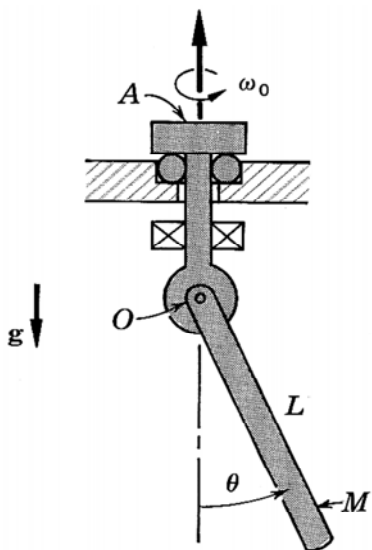
- (a) What are the forces on the bearings?
- (b) What is the kinetic energy of the rotating plate?

**Problem 4** (adapted from Crandall et al., 4-14)

The uniform rod of length  $L$  and mass  $M$  is pivoted, without friction, to the shaft OA, which revolves in fixed bearings at the steady rate  $\omega_0$ . The rod is constrained to remain in a plane through OA which rotates with the shaft.

(a) Formulate an equation of motion for  $\theta(t)$ .

(b) For each value of  $\omega_0$  there is at least one stationary angle  $\theta_0$  which the rod can maintain while steadily precessing at the rate  $\omega_0$ . Find all stationary configurations as functions of  $\omega_0$ .



**Problem 5** (adapted from Crandall et al., 4-20)

A rigid cone with apex half angle  $\alpha$  rolls steadily without slip on a horizontal surface so that it precesses about the  $Z$  axis at a constant angular rate  $\Omega$ . The cone has mass  $M$  and principal moments of inertia  $I_1$ ,  $I_1$ , and  $I_3$  at the tip.

- (a) Determine the angular momentum of the cone.
- (b) What forces and torques are required to maintain this motion?

