**2.032 DYNAMICS** Fall 2009

**Problem Set No. 7**

**Out**: Wednesday, October 21, 2009

The homework problems are for practice only. Solutions are posted in a separate file. Please work on the problems and be prepared to ask questions related to this homework in the recitation of October 27, 2009 (4:00–5:30pm in Room 1-379).

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## Problem 1

A rigid cylinder is rolling without slip inside a hollow rigid tube which is rolling without slip on the floor. The equations governing the motion of this system were derived in class. Using the same notation as in class,

(i) Verify that the governing equations of motion admit the following steady motion:

$$\phi = 0, \quad \dot{\theta} = \Omega \ \text{(constant)}.$$

(ii) Interpret this steady motion physically.

(iii) Examine the stability of the equilibrium position $\phi = \theta = 0$ to small (linear) perturbations. Solve the linearized equations of motion valid near this equlibrium, for the initial perturbation $\phi(t = 0) = \phi_0$, $\theta(0) = \dot{\theta}(0) = \dot{\phi}(0) = 0$. Interpret your solution in physical terms.

(iv) Integrate the full (nonlinear) equations of motion numerically and check the validity of the linear stability analysis in (iii) above for various values of the amplitude $\phi_0$ of the initial perturbation.

**Problem 2** (adapted from PhD Qualifying Exam 2003)

Reconsider Problem 2 of PS No. 3. Using the same notation,

(i) Derive the differential equation describing the motion of the bead on the ring.

(ii) Find equilibrium positions $\theta_0$ for the bead and investigate the stability of these positions at various speeds $\Omega$.

(iii) Draw a stability diagram showing all solution branches (and their stability properties) for $0 < \Omega < \infty$.

(iv) Draw the phase plane of solution trajectories at representative values of $\Omega$.

(v) Instead now consider a ring inclined at $120^\circ$ to the vertical, so that C is below O. *Without any calculations*, state how many equilibrium positions you expect and how their stability will vary with $\Omega$.

**Problem 3**

A rigid block of height $H$, length $L$ and depth $D$ rests on a rigid cylinder of mass $M$ and radius $R$, as shown in the sketch. The cylinder rolls on the floor without slipping and the block rolls on the cylinder without slipping as well.

(a) Study the geometry of motion of the two rigid bodies, paying attention to the rolling constraints. Select a suitable set of independent generalized coordinates to describe this motion. Show your choice of coordinates *clearly* on a sketch.

(b) Derive the governing equations of motion in terms of these coordinates.

(c) Examine the stability to small perturbations of the equlibrium position of the block.