Angular momentum with different points

I. General system

\[ H_e = \sum_i r_i \times p_i \]

\[ r_i = R_{BA} + R_i \]

\[ H_e = R_{BA} \times \sum_i \frac{p_i}{m_i} + \sum_i R_i \times \frac{p_i}{m_i} \]

\[ \Rightarrow H_B = H_A + R_{BA} \times \frac{p}{m} \quad (*) \]

Relation (*) is valid in general: for any two points \( A, B \) (not necessarily \( C \)) and for a general system, not necessarily a rigid body.

II. Rigid body

Suppose now \( A = C \) (mass center of rigid body)

\[ (*) \Rightarrow H_B = H_C + k_C \times \frac{p}{m} \]

Where

\[ H_C = \sum_i r_i \times \frac{p_i}{m_i} \]

\[ V_i = \omega \times R_i \]

\[ \Rightarrow H_C = \sum_i V_i \times R_i + \sum \frac{m_i R_i}{m_i} \times (\omega \times R_i) \]

\[ \Rightarrow H_C = [I]_C \omega \quad \text{only for rigid body and mass center} \]