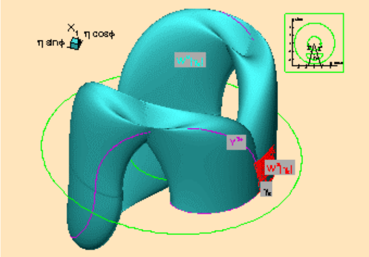
Advanced Nonlinear Dynamics and Chaos

(18.386J/2.037J)



George Haller (x2-3064, Rm. 3-354)

Mechanical Engineering

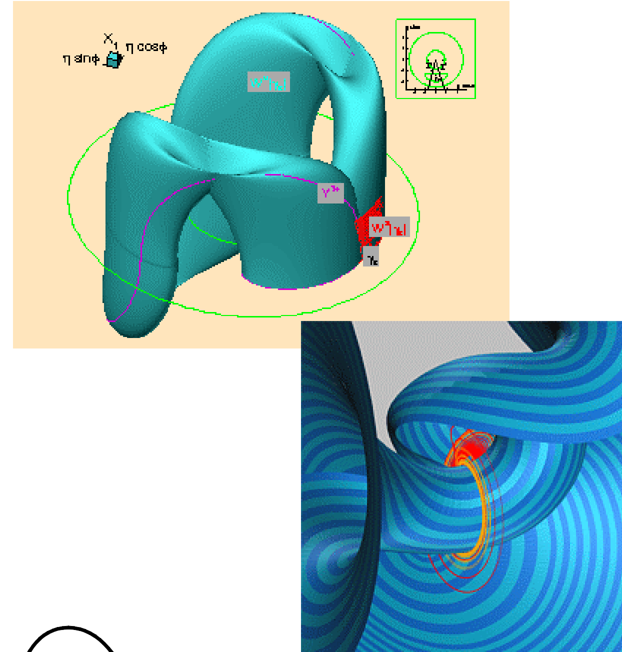
MIT

Logistics

- Lectures: Tuesday,Thursday, 11:00am-12:30pm, Room 1-242
- Office hours: Tuesdays, 3-4:30pm, Rm. 3-352
- Homeworks: - Typically every week, out on Thursday, due in a week
  - Late homework accepted if prior arrangement is made
- Report: - Written report on a research article as part of the final grade.
- Textbook: None required. Recommended books on reserve in Baker library:

  1. Guckenheimer, J., and Holmes, P., **Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields**
  2. Chicone, C., **Ordinary Differential Equations with Applications**
  3. Arnold, V. I., **Mathematical Methods of Classical Mechanics**

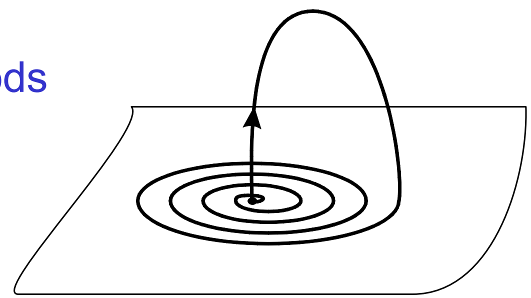
Course outline



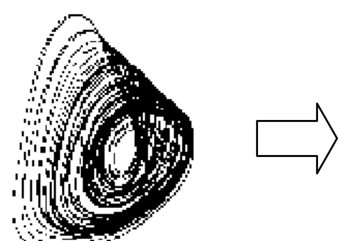
(by H. Osinga, Bristol)

- Normally hyperbolic invariant manifolds
    - Introduction to manifolds
    - Existence and persistence of invariant manifolds
    - Geometric singular perturbation theory

- Global bifurcations
    - Higher-dimensional Melnikov methods
    - Shilnikov orbits
    - Homoclinic bifurcations



- The internal structure of chaos
    - Symbolic dynamics
    - Bernoulli shift map
    - Subshifts of finite type
    - Higher-dimensional chaos



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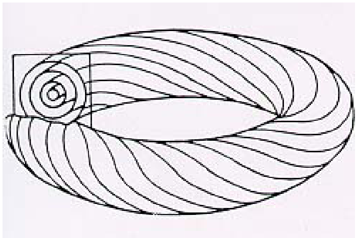
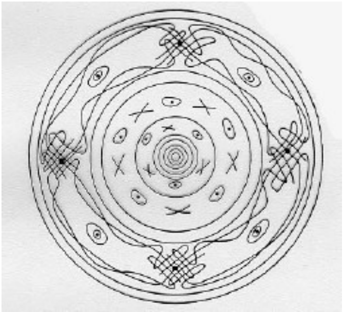
- Hamiltonian dynamical systems
  - Canonical and noncanonical Hamiltonian systems
  - Symplectic geometry
  - Conservation properties, phase space geometry

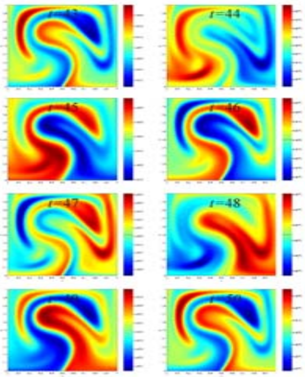
$$\dot{\mathbf{x}} = \mathbf{J}(\mathbf{x})\mathbf{D}\mathbf{H}(\mathbf{x})$$



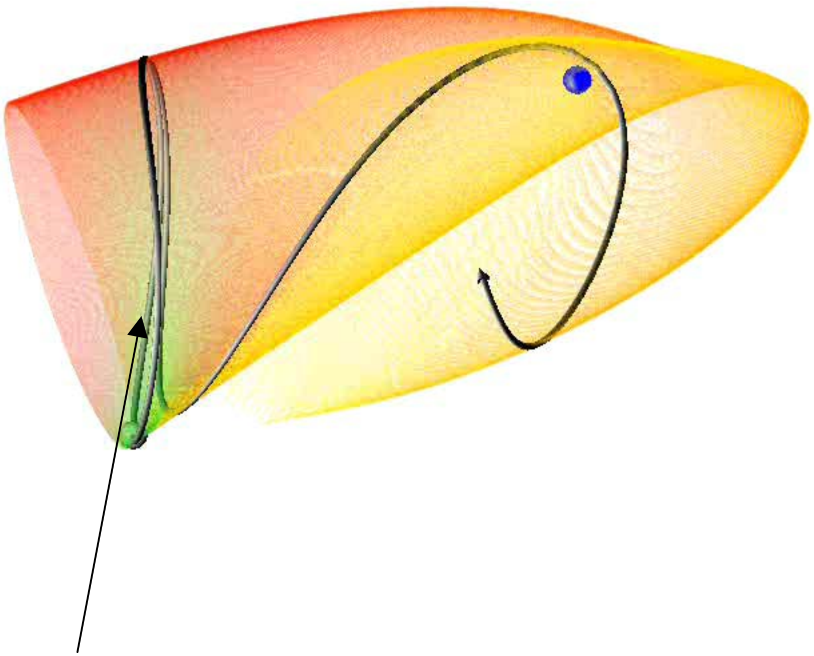
- Integrable and near-integrable systems
  - Liouville-Arnold theory: existence of invariant tori
  - KAM-theory: persistence of invariant tori
  - Arnold diffusion



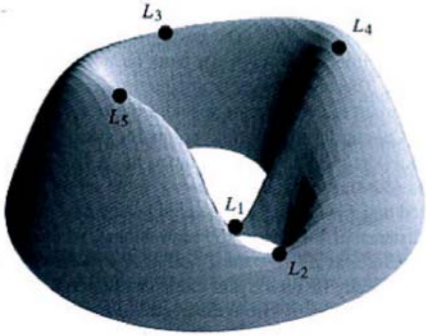
- Introduction to infinite-dimensional dynamics
  - Attractors, inertial manifolds
  - PDEs as infinite-dimensional Hamiltonian systems
  - Chaos in infinite dimensions



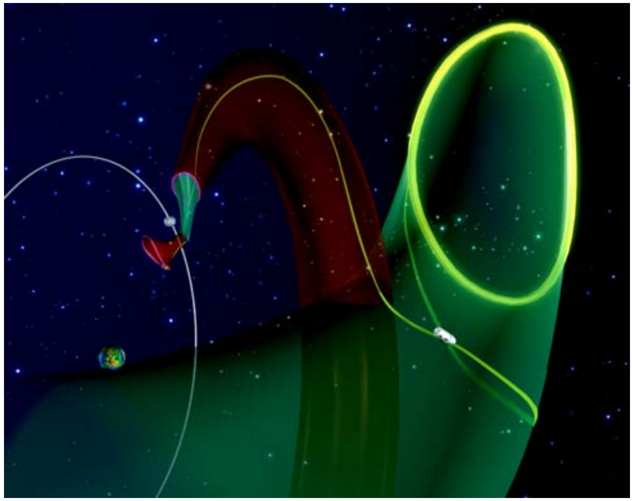
Motivational example I: Energy-efficient trajectory of a spacecraft along an unstable manifold (Caltech-JPL)



A **halo orbit** around the L1 equilibrium point in the circular restricted three body problem.
(plot by GAIO of Michael Dellnitz and Oliver Junge, Institute of Mathematics, University of Padeborn)

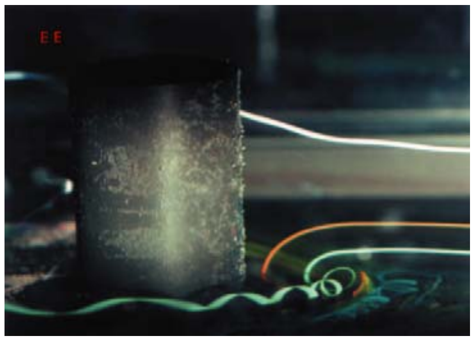


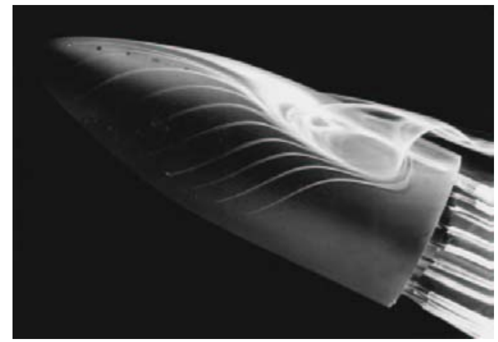
Equilibria of planar restricted 3-body problem



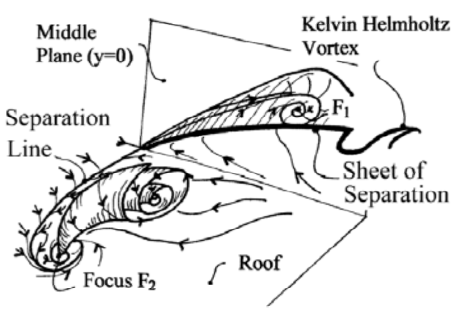
Halo hopping

Motivational example II: Unsteady fluid flow separation on no-slip surfaces

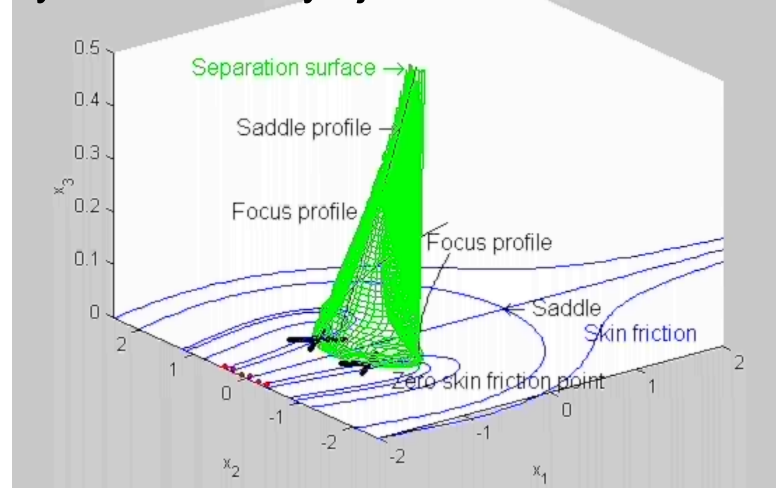




Flow separation: particles following the boundary are suddenly ejected into the flow

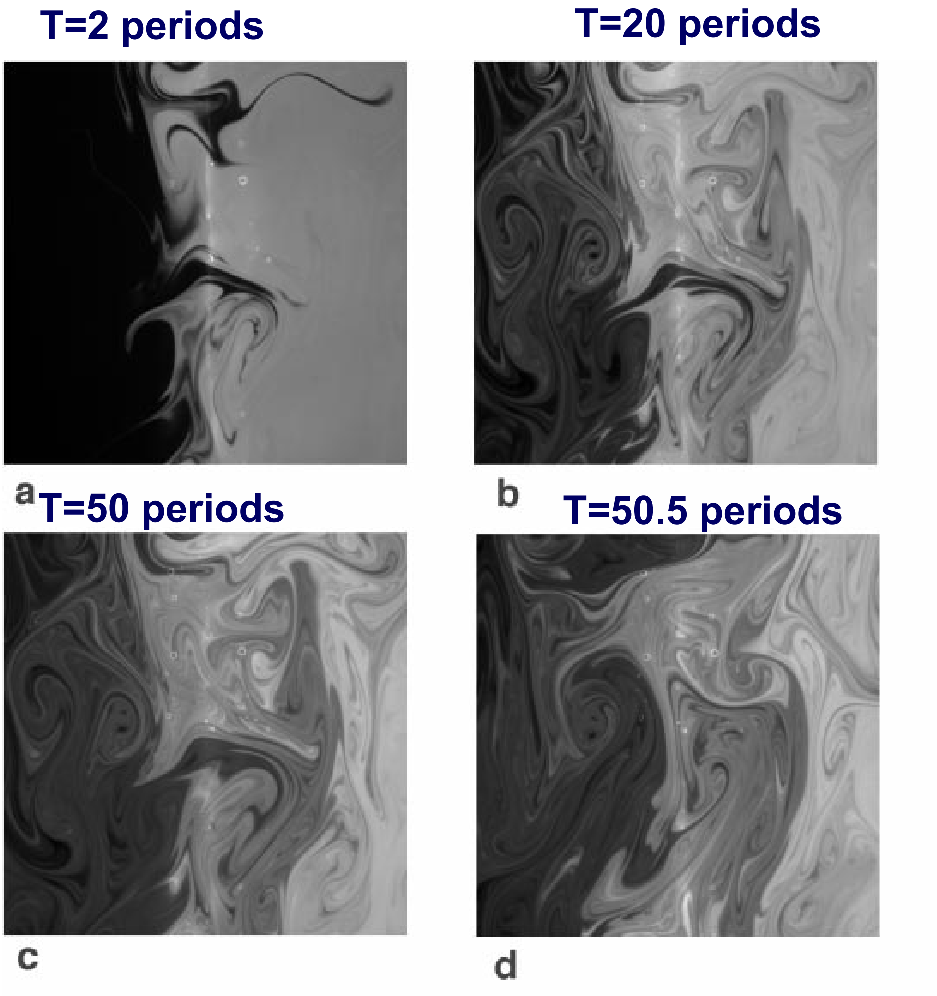


Separation on the roof of a passenger car
Gillieron & Chometon [1999]



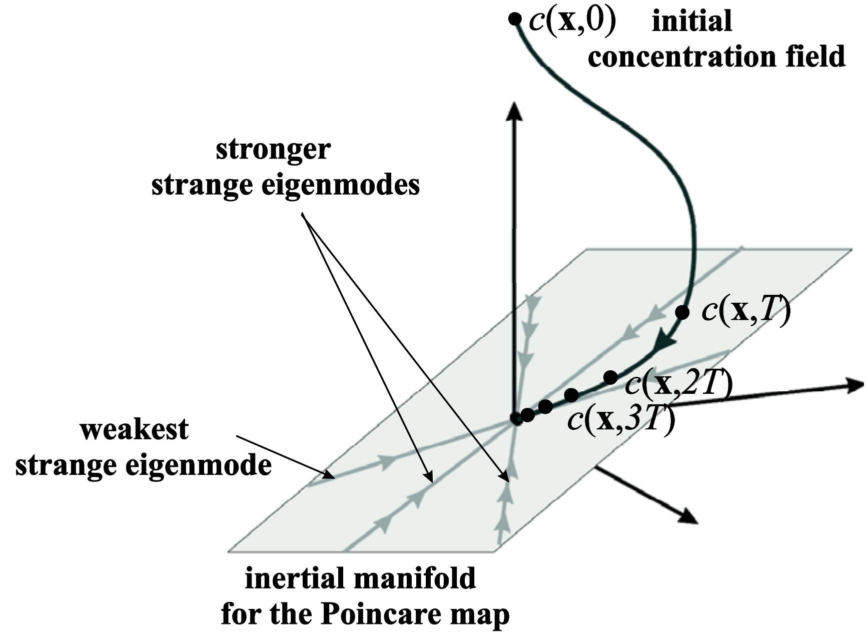
O. Grunberg [2003]

Motivational example III: Mixing of diffusive substances



Rothstein, Henry, & Gollub [1999]

In function space:



Liu & H. [2003]