MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Department of Mechanical Engineering

Course 2.05 **Kinematics and Dynamics of Mechanisms and Manipulators**

Fall 2000

Problem Set No. 4 — Assigned: 10/05/00

Due: 10/17/00

<u>Problem 4.1</u>

The well known Hooke universal joint, see Figure (1a) (for coupling misaligned shafts) is a special case of the spherical four bar mechanism. This can be seen from the equivalent mechanism shown in Figure (1b).

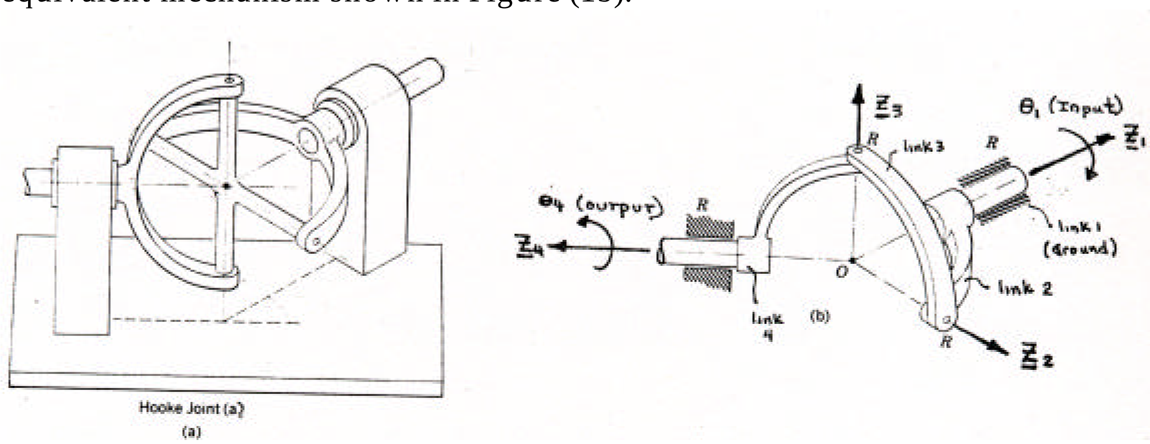

Figure 1: Hooke universal joint

For the axes shown in Figure B it can be shown that this mechanism can be represented by the Hartenberg and Denavit transformation variables given in Table I.

Table I

A_1 or $T^1{}_2$:	$\theta_1(t)$	$r_1 = 0$	$a_1 = 0$	$\alpha_1 = 90^0$
A_2 or $T^2{}_3$:	$\theta_2(t)$	$r_2 = 0$	$a_2 = 0$	$\alpha_2 = 90^0$
A_3 or $T^3{}_4$:	$\theta_3(t)$	$r_3 = 0$	$a_3 = 0$	$\alpha_3 = 90^0$
A_4 or $T^4{}_1$:	$\theta_4(t)$	$r_4 = 0$	$a_4 = 0$	$\alpha_4 = -135^0$ (the misalignment)

(a) Calculate the mobility (dof) of this device. Is the result consistent with its use? If it is not consistent, explain why?

(b) Set up an equation of 4x4 matrices which would permit the solution for θ_2, θ_3 and θ_4 knowing θ_1. Do not attempt to solve it, but be sure to find all elements in the matrices based on Table I.

(c) Explain how your equation in part (b) might be solved.

Problem 4.2

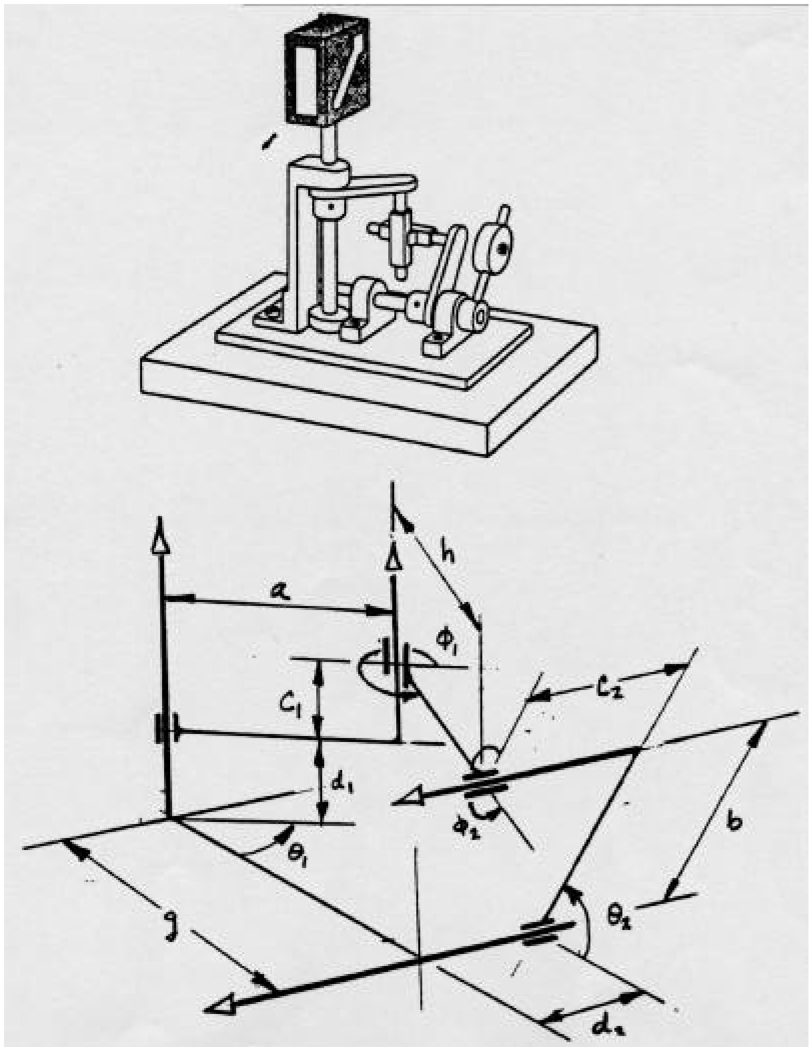

Figure 2: A railroad signal mechanism (a) and its associated diagram (b)

A railroad signal mechanism shown in Figure 2a. This mechanisms is modeled as the C4 (four cylindrical joints) mechanism shown in Figure b (two connected Dyads) . The joint parameters are $\theta 1, \theta 2$, $\phi 1, \phi 2$, c1, c2, d1, d2. Analyze this mechanism by:

1) Calculate the mobility F of this spatial mechanism

2) Attach the Denavit-Hartenberg frames to this mechanism

3) Determine its kinematic loop matrix equation using the disconnected loop method

4) Equate terms In the scalar equations and determine relations between all the joint parameters

5) Finally solve for the joint parameters In term of $\theta 1$, the rotation of the Input crank, and d1, the translation of the first joint. (Note: In the actual mechanism, d1 is held constant).

Problem 4.3

In the following figure is a robot called SCARA. The coordinate systems are given. Note that H3 is always negative and that joint angles vary from $-\pi/2$ to $+\pi/2$.

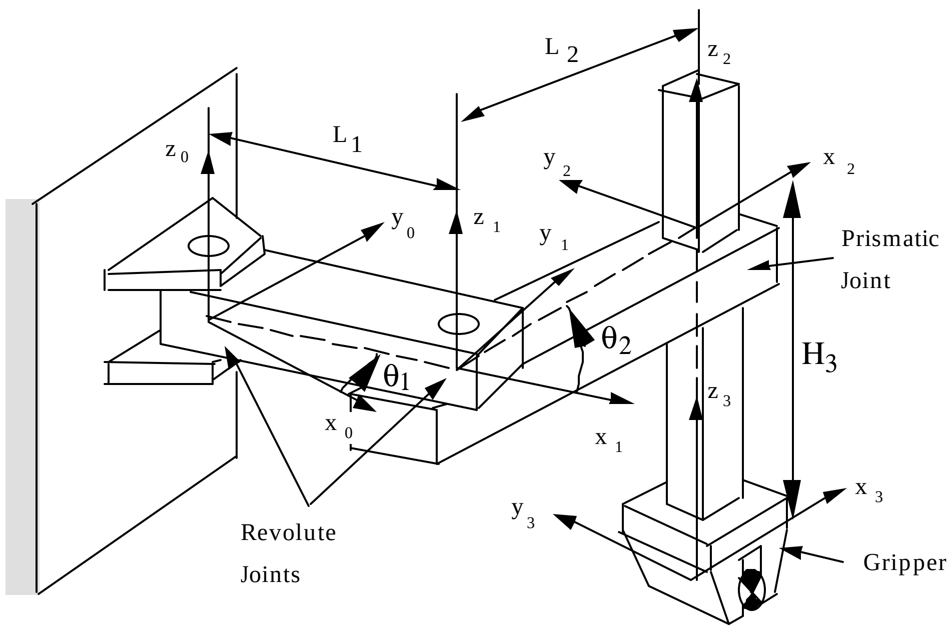

Figure 3: The 3-dof SCARA Robot

1) Determine the position and the orientation of the gripper with a simple vector analysis. The orientation will be given by coordinate of **x3**,**y3** and **z3** in (**x0,y0,z0**).

1) Identify the basic homogeneous transformation matrices and calculate the global 4x4 position / orientation matrix.

2) Write out Denavit-Hartenberg parameters for each link, the associated 4x4 matrices and calculate the global position / orientation matrix.

3) What kind of spatial transformations this kind of manipulator can generate? Describe the operational space in translation. How is related the orientation?

4) A position task is given: $x_d=L_1/2$, $y_d=L_1*\sqrt{(3/4)}+L_2$, $z_d=-(L_1+L_2)/4$
Give all the solutions in terms of joint values (θ_1, θ_2, H_3) that achieve this task.
Give the orientation for this task in terms of the Euler angles (convention ZYX).

Problem 4.4

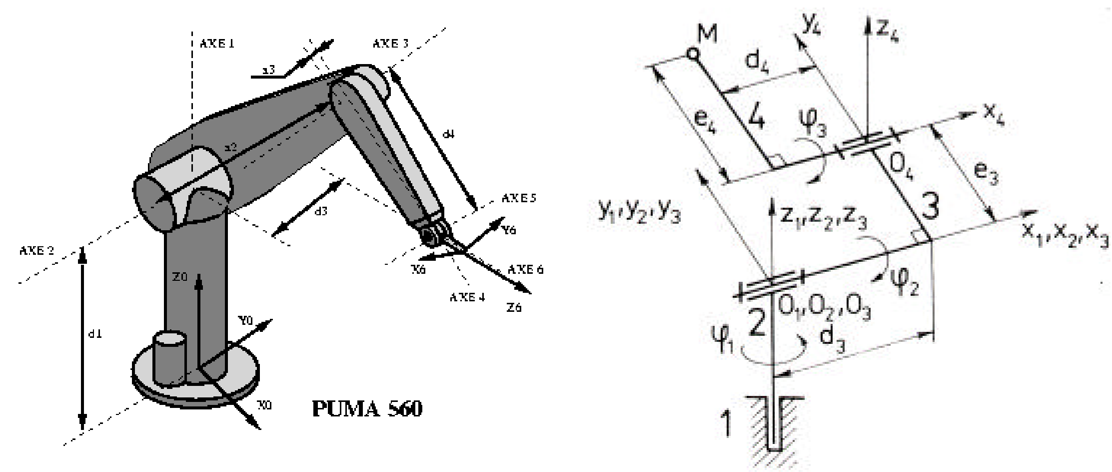

Figure 4: The PUMA 560 (a) and an associated kinematic diagram (b)

The PUMA 560 (Figure 4a) is a very popular robot in robotics laboratories as well as in industry. For this problem, you will use the data given in the diagram (Figure 4b). Do not include transformations or variables that are not shown in the diagram. Here, only the first three degrees of freedom (revolute joints φ1,φ2,φ3) are considered.

1) Identify all the basic transformations and write out their associated 4x4 matrices

2) Determine the global 4x4 matrix then give the global coordinates of point M and the orientation the last frame (attached to link 4) according to the Euler ZYX angle convention.

3) Explain the nature and the limitations of the operational space (or workspace). Why this part of the manipulator is called "the positioning structure"? Could it be used to completely orient the end effector in 3D space?

4) We want to add an orienting structure to this manipulator. We propose to add a wrist composed by three revolute joints with intersecting axis (spherical joint). Explain the advantages of this wrist on the manipulator kinematics problem.

5) Propose a general method to solve the inverse kinematics problem using the wrist advantage. Considering that the wrist has the same rotation matrix than the Euler ZYX convention angle (given in class) and has no effect on the position, apply your method to solve the inverse kinematics for the following (simple) task :

Position: Xd=d3-d4 , Yd=e3*√(1/2) , Zd=e4 - e3*√(1/2)

Orientation (Euler ZYX) : $\alpha = 0$, $\beta = \pi/4$, $\gamma = \pi/2$