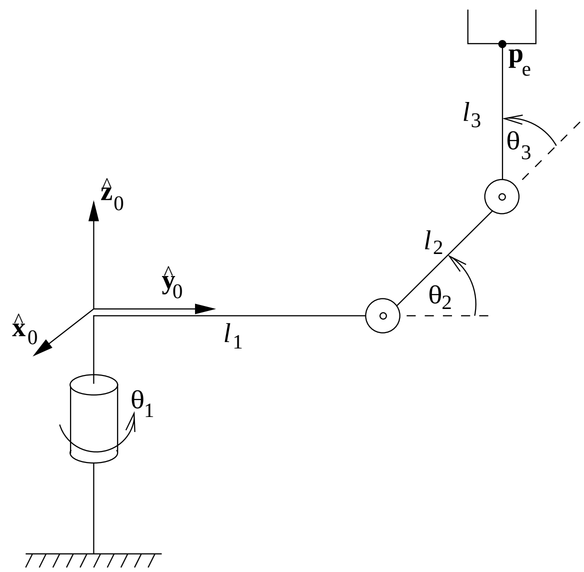
**Problem 2:**
For the robot diagrammed in this Figure:




(A) Derive the Jacobian $\mathbf{\Gamma}$, such that

$$\frac{d\mathbf{p}_{0\mathrm{e}}}{dt} = \begin{bmatrix} \dot{x}_{0\mathrm{e}} \\ \dot{y}_{0\mathrm{e}} \\ \dot{z}_{0\mathrm{e}} \end{bmatrix} = \mathbf{\Gamma} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}$$

Note that the $i$-th column-vector of this Jacobian represents the Cartesian velocity of the end-effector $\dot{\mathbf{p}}_{0\mathrm{e}}$ generated by a unit rotational velocity in joint $i$ ($\dot{\theta}_i = \omega_i = 1$). You already know how to compute this using the vector cross-product:

$$\frac{d\mathbf{p}_{0\mathrm{e}}}{d\theta_i} = \hat{\boldsymbol{\omega}}_i \times \mathbf{r}_{i,\mathrm{e}},$$

where $\mathbf{r}_{i,\mathrm{e}}$ is the radius for the motion of the end-effector $\mathbf{p}_{0\mathrm{e}}$ about the $i$-th joint axis, and $\hat{\boldsymbol{\omega}}_i$ is the unit vector representing a unit-speed rotation about joint axis $i$ in the positive direction.

(B) Given $l_1 = 3$, and $l_2 = l_3 = 1$, (i) describe the set of points in space that the manipulator can reach (the "reachable workspace"). (ii) What geometric shape describes the set of reachable points in Cartesian space? (iii) Which positions in the reachable workspace can we reach with more than one angular orientation of the end-effector, which positions with more than two orientations? (iv) What portion of the reachable workspace is also "dextrous workspace"? **Hint:** think about the mechanism, don't write kinematic equations.

(C) Given the same link lengths as in (B), (i) describe the configurations of the robot corresponding to workspace boundary singularities and workspace interior singularities. (ii) Verify that the interior and boundary singularities cause the Jacobian to become singular ($|\mathbf{\Gamma}| \rightarrow 0$). (iii) Describe the set of end-effector points in the workspace at which these interior and boundary kinematic singularities occur.

(D) Given the same link lengths as in (B), and robot configuration $\theta_1 = 0$, $\theta_2 = \theta_3 = \frac{\pi}{4}$, what joint torque vector $\boldsymbol{\tau} = [\tau_1 \quad \tau_2 \quad \tau_3]^T$ ($\tau_i$ is the joint torque about joint $i$) should the robot apply to generate a force vector $\mathbf{f}_{\mathrm{0e}} = [1 \quad -1 \quad 0]^T$ at the end-effector?