MASSACHUSETTS INSTITUTE OF TECHNOLOGY

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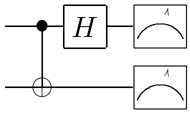
Quantum Information Science I

September 30, 2010

Problem Set #4
(due in class, 07-Oct-10)

1. **Measurement in the Bell basis** Show that the circuit



performs a measurement in the basis of the Bell states. Specifically, show that this circuit results in a measurement being performed with four operators $\{M_k\}$ such that $M_k^\dagger M_k$ are the four projectors onto the Bell states.

2. **Schmidt numbers and LOCC**. Recall that the Schmidt number of a bi-partite pure state is the number of non-zero Schmidt components. Prove that the Schmidt number of a pure quantum state cannot be increased by local unitary transforms and classical communication. The Schmidt number is strictly nonincreasing under more general conditions, namely, for *arbitrary* local operations and classical communication (LOCC); you are welcome to prove this also, but that is not required for credit. The Schmidt number is one measure of how entangled a bi-partite quantum state is.

3. **Reversible circuits**.

   (a) Construct a reversible circuit which, when two bits x and y are input, outputs $(x, y, c, x \oplus y)$, where c is the carry bit when x and y are added.

   (b) Construct a reversible circuit using Fredkin gates to simulate a Toffoli gate.

   (c) Construct a quantum circuit to add two two-bit numbers x and y modulo 4. That is, the circuit should perform the transformation $|x, y\rangle \to |x, x + y \bmod 4\rangle$.

4. **Quantum circuit for the Hamming weight**. Construct a quantum circuit that performs the following unitary transformation:

$$|z\rangle|0\rangle \to |z\rangle|w(z)\rangle \,,$$

where $w(z)$ denotes the Hamming weight of z (the number of ones in its binary representation). Try to do this for the general case of z being represented by n qubits, for arbitrary n, but if you cannot think of a clever solution (which takes avantage of quantum gates, versus just classical ones), just give a circuit for $n = 3$.