2.111J/18.435J Quantum Computation Final Exam
(Given 9 AM – 12 Noon, with 2 pages of notes allowed, on Wednesday, December 21, 2005)

(0) What Is a Tensor? (*9 points*)

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(1) Buzzwords. Fill in the blank with appropriate word or phrase. (*2 points apiece*)

(1a) 'Harry and are perfect for each other. We seem to know more about each other's feelings than is humanly possible. We're like two ____________ qubits.'

(1b) 'I'm so confused, I can't figure out which direction is up and which is down. I'm like a nuclear spin in a coherent ____________ of $|0\rangle$ and $|1\rangle$.'

(1c) A ____________ gate and a ____________ gate are both reversible logic gates. The first preserves the number of zeros and ones in the input, the second does not.

(1d) 'Jeanette is so logical, it can be frustrating. Sometimes I think that she wants to reduce our whole relationship to a calculation in ____________ ____________ .'

(1e) The ____________ state and the ____________ state both exhibit quantum weirdness. But the former has only two qubits while the latter has three.

(1f) Quantum computation poses a threat to conventional ____________ cryptography.

(1g) 'You and I are looking at this relationship from completely complementary points of view. It's as if I'm position and you're momentum. If we shared a bunch of qubits, for you to see things my way, you would have to perform a ____________ ____________ ____________ on them.'

(1h) A nuclear spin can't do a belly-flop, but it can do a ____________ ____________.

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(2) Magnetic Resonance. (*25 points*)

A proton is sitting in a static magnetic field of 10.000 Tesla, oriented along the z-axis. You can apply an oscillating microwave field to the proton. The strength of the oscillating field is 0.100 Tesla. You can choose both the frequency of the applied field and the duration of the microwave pulse to be applied. The proton is initially oriented with its spin up along the z-axis. Its magnetic moment is 1.410607×10^{-26} Joules/Tesla. (Another fundamental constant you may wish to note is $\hbar = 1.054573 \times 10^{-34}$ Joules $\cdot$ sec.)

(2a) Your goal is to rotate the proton spin by an angle π about the axis $(\hat{x} + \hat{z})/\sqrt{2}$ in the co-rotating frame. This action can be performed by applying a single microwave pulse. What frequency should the pulse have?

(2b) For what duration should the pulse be applied?

(2) Magnetic Resonance (*continued*).

(2c) Discuss how the effect of the pulse applied in (2a-b) resembles and differs from an idealized Hadamard gate.

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(3) Josephson Junction Hamiltonian. (*25 points*)

The Hamiltonian for a Josephson junction system is $H = P^2/2 - \cos X$. Here, P and X are operators that are analogous to momentum and position: X correponds to the phase Φ of the supercurrent across the junction, and P corresponds to $d\Phi/dt$. Note that in the vicinity of $|x = 0\rangle$, this Hamiltonian is close to that of an harmonic oscillator. As x moves away from 0, the operator becomes anharmonic. The Hamiltonian H is in fact the Hamiltonian for a *physical pendulum*, in other words, a rigid rotor acted on by a constant force.

(3a) Using the commutation relation, $[X, P] = i\hbar$, find formulae for $[X, P^2]$ and $[P, X^2]$.

(3b) Find formulae for $[X, P^n]$ and $[P, X^n]$.

(3c) In the Heisenberg picture of quantum mechanics, one looks at the time evolution of *operators* such as X and P, rather than the time evolution of states such as $|x\rangle$ and $|p\rangle$. The time evolution of an operator A is given by $dA/dt = (i/\hbar)[H, A]$. Using the results of (3a-b), give expressions for dX/dt and dP/dt.

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(4) Decoherence Free Subspace. (*25 points*)

A collective error is one in which the same error occurs to each qubit. For example, if there are four qubits, a collective error could be of the form $U \otimes U \otimes U \otimes U$, where $U = e^{-i\theta\sigma/2}$ is a rotation by θ about an axis determined by σ. A four qubit quantum code to correct collective errors is defined as follows:

$$|0_C\rangle \rightarrow \frac{1}{2}(|01\rangle - |10\rangle) \otimes (|01\rangle - |10\rangle)$$

$$|1_C\rangle \rightarrow \frac{1}{\sqrt{12}}(2|1100\rangle + 2|0011\rangle - |1010\rangle - |0101\rangle - |0110\rangle - |1001\rangle).$$

(4a) Calculate the effect of an error $U \otimes U \otimes U \otimes U$ on the encoded version of the state $\alpha|0\rangle + \beta|1\rangle$, first for $U = \sigma_z$ and then for $U = e^{-i\theta\sigma_z/2}$. Show your work.

(4b) Calculate the effect of $U \otimes U \otimes U \otimes U$, where $U = e^{-i\theta\sigma_x/2}$.

(4c) Calculate the effect of $U \otimes U \otimes U \otimes U$, where $U = e^{-i\theta\sigma_y/2}$.

(4d) What is the effect of $U \otimes U \otimes U \otimes U$, for a generic $U = e^{-i\theta\sigma/2}$?