

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

2.111J/18.435J/ESD.79

Quantum Computation

Fall 2004

Problem Set 10

Optional (not to be turned in)

Problem 1. Single-qubit σ_X errors can project the codeword $\alpha|000\rangle + \beta|111\rangle$ onto one of the following subspaces: $\{|000\rangle, |111\rangle\}$, $\{|100\rangle, |011\rangle\}$, $\{|010\rangle, |101\rangle\}$, and $\{|001\rangle, |110\rangle\}$. Construct a quantum circuit that specifies in which subspace the received codeword is. You can use two work qubits, some operations on the work qubits and the original qubits, and finally, a measurement on the work space.

Problem 2. Show how to correct a single σ_Z error for the phase-error correcting code:

$$|0\rangle \rightarrow \frac{1}{2}(|000\rangle + |110\rangle + |101\rangle + |011\rangle)$$

$$|1\rangle \rightarrow \frac{1}{2}(|111\rangle + |001\rangle + |010\rangle + |100\rangle).$$

Problem 3. For the Shor's nine-qubit code:

$$|0\rangle \rightarrow \frac{1}{2}(|000\rangle|000\rangle|000\rangle + |000\rangle|111\rangle|111\rangle \\ + |111\rangle|000\rangle|111\rangle + |111\rangle|111\rangle|000\rangle)$$

$$|1\rangle \rightarrow \frac{1}{2}(|000\rangle|000\rangle|111\rangle + |000\rangle|111\rangle|000\rangle \\ + |111\rangle|000\rangle|000\rangle + |111\rangle|111\rangle|111\rangle),$$

give a quantum circuit that corrects a possible single-qubit Pauli error.

Problem 4. For the quantum Hamming code, show that the vector $|-\rangle = (|0\rangle - |1\rangle)/\sqrt{2}$ gets encoded to

$$\frac{1}{4} \left(\sum_{x \in \{H\}} |x\rangle - \sum_{x \in \{G\} - \{H\}} |x\rangle \right),$$

where $\{H\}$ is the corresponding binary subspace spanned by

$$H = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

and $\{G\} - \{H\}$ is the set of complements of $\{H\}$, which is all elements of $\{H\} + [1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1]$.