PS3 Solutions

1 (exercise 2.59)

$$\langle X \rangle = \langle 0|X|0\rangle = \langle 0|1\rangle = 0$$

$$\langle X^2 \rangle = \langle 0|X^2|0\rangle = \langle 0|X|1\rangle = \langle 0|0\rangle = 1$$

$$\Delta(X) = \langle X^2 \rangle - \langle X \rangle^2 = 1$$

2 (exercise 2.60)

Calculate eigenvalues directly from $v \cdot \sigma$. Have to prove $v \cdot \sigma P_{\pm}|\psi\rangle = \pm P_{\pm}|\psi\rangle$. Use fact that $(v \cdot \sigma)^2$ is 1 (also, using this fact, by applying $v \cdot \sigma$ twice to one of its eigenvectors, it follows that the square of the eigenvalues is 1)

3 (exercise 2.61)

State of the system is the eigenvector of $v \cdot \sigma$ with eigenvalue one. The probbaility is the modulus square of the inner product between that vector and $|0\rangle$.

4 (exercise 2.69)

Just algebra.

5 (exercise 2.70)

Show that $\langle\psi|E \otimes I|\psi\rangle$ has the same value for any of the Bell states, hence Eve learns nothing.

6 (exercise 2.72)

(1) $\rho = \sum p_i|\psi_i\rangle\langle\psi_i|$; $r \cdot \sigma$ is the set of all traceless Hermitian matrices; So basically you have to prove that $2\sum p_i|\psi_i\rangle\langle\psi_i| - I$ is a traceless Hermitian matrix, and that $(2\sum p_i|\psi_i\rangle\langle\psi_i| - I)^2 \leq 1$ (because this quantity is $(r \cdot \sigma) = ||r||^2$)
(2) $r = 0$
(3) 1. If ρ is pure, then prove $(2|\psi\rangle\langle\psi| - I)^2 = 1$.
2. If $(2\sum p_i|\psi_i\rangle\langle\psi_i| - I)^2 = 1$, then prove all p_i's are 0 except 1.
(4) Just show the angle correspondence

7 (exercise 2.79)

Just use the method from the proof of the Schmidt decomposition.

8 (exercise 2.80)

$$|\psi\rangle = \sum \lambda_i |i_A\rangle |i_B\rangle; \quad |\phi\rangle = \sum \lambda_j |j_A\rangle |j_B\rangle$$

and the λ's are identical.
Chose unitary operators U and V shich that U maps $|i_A\rangle$'s into $|j_A\rangle$'s and U maps $|i_B\rangle$'s into $|j_B\rangle$'s

9 (exercise 4.5)

Just write the matrix and compute the square.

10 (exercise 4.6)

From ex. 2.72: $\lambda \cdot \sigma = 2|\psi\rangle\langle\psi| - I$. Prove that if $|\psi\rangle$ goes into $R_n(\theta)|\psi\rangle$, then $\lambda \cdot \sigma$ goes into $R_n(\theta)(\lambda \cdot \sigma)R_n(-\theta)$ and see what this means for λ.

11 (exercise 4.7)

Expand $XR_y(\theta)X$ as $X(a + b\theta Y)X = a - b\theta Y = R_y(-\theta)$