2.111J/18.435J Quantum Computation Problem Set 1
(Due: Tuesday, September 20, 2005)

1) Draw how to implement the NAND gate using a combination of capacitors, pFETs, and/or nFETs.

Notes: NAND is the Boolean logic gate which takes two binary inputs X and Y and outputs 0 if $X = Y = 1$ and otherwise outputs 1. Capacitors store bit values: 0 volts = "0" and +5 volts = "1". A pFET is a transistor switch which allows current to pass through it if there's 0 volts on its gate and does not allow current to pass through it if there is +5 volts on its gate. Conversely, a nFET is a transistor switch which allows current to pass through it if there is +5 volts on its gate and doesn't allow current to pass through it if there is 0 volts on its gate.

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2) Verify that AND is a nonlinear gate.

Notes: In other words, verify that if we denote the inputs to AND by the letters X and Y, then there does not exist any function of the form $aX + bY + c$ (a, b, c are constants) that reproduces the truth table of X AND Y.

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3) Prove that all reversible gates with 2 inputs and 2 outputs are linear.

Notes: In other words, prove that any 2 input / 2 output gate that is reversible and has inputs X and Y must have outputs that are expressible in the form $aX + bY + c$ and $a'X + b'Y + c'$ where a, a', b, b', c, c' are constants.

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4) Prove that the Fredkin (controlled-SWAP) gate is universal by explicitly showing how to make AND, OR, NOT, and COPY gates out of one or more Fredkin gates.

Notes: The Fredkin (controlled-SWAP) gate is a gate that takes three inputs X, Y, and Z and produces three outputs X', Y', and Z' according to the rule that if $X = 0$ then it does nothing (*i.e.,* $X' = X, Y' = Y, Z' = Z$) and if $X = 1$ then it swaps Y and Z (*i.e.,* $X' = X, Y' = Z, Z' = Y$). In making AND, OR, NOT, and COPY, you are free to specify some gate inputs to be fixed values.