MASSACHUSETTS INSTITUTE OF TECHNOLOGY

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Quantum Computation
Fall 2004

MIDTERM EXAM
Thursday, October 28

Problem 1. In NMR quantum computing, a Hadamard gate is implemented by rotating around the axis $(\vec{x} + \vec{z})/\sqrt{2}$. Compute the matrix obtained by rotation around this axis by π radians, and compare to a Hadamard gate.

Problem 2. Let

$$H = \frac{1}{2}(\sigma_X \otimes \sigma_X + \sigma_Y \otimes \sigma_Y + \sigma_Z \otimes \sigma_Z + I \otimes I)$$

be an operator on two qubits.

a) Find H^2 and write it in a simple form.
b) Using (a), find $\exp(-i\pi H/4)$ and $\exp(-i\pi H/2)$.
c) Find the eigenvalues of H.
d) Find a set of orthonormal eigenstates of H.

Problem 3. Let N be an integer larger than 5. Consider the following state:

$$|\psi\rangle = \frac{1}{\sqrt{N}} \sum_{x=0}^{N-1} |x \bmod N\rangle_A \otimes |3x \bmod N\rangle_B \otimes |5x \bmod N\rangle_C .$$

Find the output state if we take a quantum Fourier transform modulus N on each of the registers *A*, *B*, and *C*. That is, if we denote the corresponding QFT operators to each system by U_A, U_B, and U_C, find $U_A \otimes U_B \otimes U_C |\psi\rangle$. Write your answer in the basis $\{|i\rangle_A |j\rangle_B |k\rangle_C \mid 0 \le i, j, k < N\}$, and show that it is the superposition of equally probable states. What is this probability?