Problem 1. In NMR quantum computing, a Hadamard gate is implemented by rotating around the axis \((\vec{x} + \vec{z}) / \sqrt{2}\). Compute the matrix obtained by rotation around this axis by \(\pi\) radians, and compare to a Hadamard gate.

Problem 2. Let
\[
H = \frac{1}{2} (\sigma_X \otimes \sigma_X + \sigma_Y \otimes \sigma_Y + \sigma_Z \otimes \sigma_Z + I \otimes I)
\]
be an operator on two qubits.

a) Find \(H^2\) and write it in a simple form.

b) Using (a), find \(\exp(-i\pi H / 4)\) and \(\exp(-i\pi H / 2)\).

c) Find the eigenvalues of \(H\).

d) Find a set of orthonormal eigenstates of \(H\).

Problem 3. Let \(N\) be an integer larger than 5. Consider the following state:
\[
|\psi\rangle = \frac{1}{\sqrt{N}} \sum_{x=0}^{N-1} |x \mod N\rangle_A \otimes |3x \mod N\rangle_B \otimes |5x \mod N\rangle_C
\]
Find the output state if we take a quantum Fourier transform modulus \(N\) on each of the registers \(A\), \(B\), and \(C\). That is, if we denote the corresponding QFT operators to each system by \(U_A\), \(U_B\), and \(U_C\), find \(U_A \otimes U_B \otimes U_C |\psi\rangle\). Write your answer in the basis \(\{|i\rangle_A |j\rangle_B |k\rangle_C \mid 0 \leq i,j,k < N\}\), and show that it is the superposition of equally probable states. What is this probability?