2.111J/18.435J/ESD.79 Quantum Computation — Due: September 18, in class

Problem Set 1

Problem 1

Construct the OR gate out of AND, NOT and COPY gates. You can choose to use inputs '0' and '1', extra credit for doing without.

Problem 2

Are CNOT and CCNOT reversible?

Problem 3

Can you construct AND, OR, NOT and COPY out of (a)CNOT? (b) of CCNOT? Once again, you may use '0' and '1' additional inputs.

Problem 4

Define the Pauli spin matrices to be the following 2×2 matrices:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

We note that these matrices are hermitian, in other words $\sigma_i = \sigma_i^\dagger$, for i=x, y and z.

1. Show: $\sigma_x^2 = \sigma_y^2 = \sigma_z^2 = I$, where I is the 2×2 identity matrix. Note that since the Pauli matrices are hermitian, this would imply that they are also unitary.

2. Let $[\sigma_x, \sigma_y] \equiv \sigma_x\sigma_y - \sigma_y\sigma_x$ be the commutator of σ_x and σ_y. Show: $[\sigma_x, \sigma_y] = 2i\sigma_z$ and the cyclic permutations of this, i.e. $[\sigma_z, \sigma_x] = 2i\sigma_y$, $[\sigma_y, \sigma_z] = 2i\sigma_x$.

3. Let $\vec{\sigma} = \hat{i_x}\sigma_x + \hat{i_y}\sigma_y + \hat{i_z}\sigma_z$ be the generalized Pauli matrix about the $\hat{v}$ axis. Here we take the vector $\hat{v} = (i_x\ i_y\ i_z)$ to be normalized to 1. Show: $\vec{\sigma}^2 = I$, where once again I is the 2×2 identity matrix.

4. Using the Taylor's expansion series $\exp x = 1 + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$ (still true with matrices for argument of the exponent!) together with $\vec{\sigma}^2 = I$, show:

$$\exp\left(-i\frac{\theta}{2}\sigma.\hat{v}\right) = I\cos\left(\frac{\theta}{2}\right) - i(\vec{\sigma}.\hat{v})\sin\left(\frac{\theta}{2}\right)$$