2.111J/18.435J/ESD.79 Quantum Computation — Due: September 25, in class

Problem Set 2

Problem 1

What are the eigenvalues and corresponding eigenvectors of σ_y? Label the eigenvector of the larger eigenvalue $|\bigotimes\rangle$ and the other $|\bigodot\rangle$.

Problem 2

Check: $e^{-i\frac{\pi}{4}\sigma_y}\,|\uparrow\rangle = k_1\,|\rightarrow\rangle$, where $|\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $|\rightarrow\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$, and k_1 is a constant to be determined.

Problem 3

1. Check: $e^{-i\frac{\pi}{4}\sigma_z}\,|\rightarrow\rangle = |\bigotimes\rangle$,
2. Check: $e^{+i\frac{\pi}{4}\sigma_z}\,|\rightarrow\rangle = |\bigodot\rangle$, and

where $|\bigotimes\rangle$ and $|\bigodot\rangle$ are the eigenvectors of σ_y found in Problem 1.

Problem 4

Check: $e^{-i\frac{\pi}{2}\sigma_{\hat{j}}}\,|\uparrow\rangle = k_2\,|\rightarrow\rangle$, where $\hat{j} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$. Find k_2.

Problem 5

Prove that $\rho^2 = \rho$ iff ρ is a pure state.