2.111J/18.435J/ESD.79 Quantum Computation	Due: October 11, in class

Problem Set 4

Problem 1

Quantum teleportation is a protocol that allows Alice to send a normalized state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ to Bob using 1-ebit (entangled bit) and 2 c-bits (classical bits). This problem walks you through the calculations necessary to see how this protocol works.

1. Our scheme requires that Alice and Bob shares a maximally entangled pair beforehand, like say one of the Bell states $|\mathrm{B}_{00}\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B)$. Draw a 2-qubit circuit diagram that would give an output that is $|\mathrm{B}_{00}\rangle$. You are allowed to use inputs '1' and '0'. Then write down the other 3 Bell states and draw the circuits that would give you those states.
2. Alice then makes a Bell state measurement on $|\psi\rangle$ and her half of the shared entangled state. Show that she gets $|\mathrm{B}_{00}\rangle$, $|\mathrm{B}_{01}\rangle$, $|\mathrm{B}_{10}\rangle$ or $|\mathrm{B}_{11}\rangle$ with equal probabilities.
3. Alice sends the results of her measurement to Bob (classically). She does this using 2-cbits that contains the two indexes identifying the Bell state that has been measured. What should Bob do once he receives the 2-cbits from Alice, such that he will obtain $|\psi\rangle$?
4. Draw the circuit diagram that illustrates how quantum teleportation works.

Problem 2

Using the Hadamard gate and CNOT gate, construct a 3-qubit circuit to create the Greenberger-Horne-Zeilinger (GHZ) state, $|\psi\rangle_{\mathrm{GHZ}} = \frac{1}{\sqrt{2}}(|000\rangle - |111\rangle)$

Problem 3

Show $\sigma_x \otimes \sigma_x \otimes \sigma_x |\psi\rangle_{\mathrm{GHZ}} = -|\psi\rangle_{\mathrm{GHZ}}$

Problem 4

1. Show $\sigma_x \otimes \sigma_y \otimes \sigma_y |\psi\rangle_{\mathrm{GHZ}} = |\psi\rangle_{\mathrm{GHZ}}$
2. Find $\sigma_y \otimes \sigma_x \otimes \sigma_y |\psi\rangle_{\mathrm{GHZ}}$
3. Find $\sigma_y \otimes \sigma_y \otimes \sigma_x |\psi\rangle_{\mathrm{GHZ}}$