2.111J/18.435J/ESD.79 Quantum Computation — Due: Nov 8, in class

Problem Set 6

Problem 1

In the Shor algorithm we have a function $f(a)$ with $0 \leq a \leq q-1$ and $f(a+r) = f(a)$. After Fourier transforming, we end up in the state

$$\frac{1}{q}\sum_{a=0}^{q-1}\sum_{c=0}^{q-1}\exp(\frac{2\pi i a c}{q})\,|c\rangle\,|f(a)\rangle\ . \tag{1}$$

What is the probability that if you measure the registers you get a value c in the first register and a value $f(k)$ in the second?

How do you turn repeated measurements into a determination of the period r?

Problem 2

An infinite line

$$\langle a|\,H\,|b\rangle = -1 \text{ iff } a = b \pm 1\ . \tag{2}$$

I will call the energy eigenstates $|\theta\rangle$, $-\pi < \theta < \pi$, where

$$\begin{aligned} H\,|\theta\rangle &= -2\cos\theta\,|\theta\rangle\,, \text{ and} \\ \langle a|\theta\rangle &= e^{i\theta a}\ . \end{aligned}$$

Consider the state

$$|\Psi(0)\rangle = N\int_{-\pi}^{\pi} \mathrm{d}\theta e^{-(\theta-\frac{\pi}{2})^2/\Delta^2}\,|\theta\rangle\ \ , \tag{3}$$

with $\Delta \ll 1$. N is the normalization factor. Calculate $\langle a|\,e^{-iHt}\,|\Psi(0)\rangle$, and show that $e^{-iHt}\,|\Psi(0)\rangle$ is a right moving packet with speed 2. (Hint: Use $\Delta \ll 1$ to do the integral.)