2.111J/18.435J/ESD.79 Quantum Computation Due: Nov 20, in class

Problem Set 7

Problem 1

With $\hbar = 1$, define a hamiltonian, $H(t)$, that describes an electron in a magnetic field, $\vec{B(t)} = (B\cos\omega t, B\sin\omega t, B_0)$,

$$H = -\frac{\gamma}{2}(B_0\sigma_z + B(\sigma_x \cos\omega t + \sigma_y \sin\omega t)) \ . \tag{1}$$

Let $\sigma_\pm = \frac{1}{2}(\sigma_x \pm i\sigma_y)$. Show that

$$H(t) = -\frac{\gamma}{2}(B_0\sigma_z + B(\sigma_+ \mathrm{e}^{-i\omega t} + \sigma_- \mathrm{e}^{+i\omega t})) \tag{2}$$

Problem 2

Show that

$$\mathrm{e}^{\frac{i\omega t}{2}\sigma_z}\sigma_\pm \mathrm{e}^{-\frac{i\omega t}{2}\sigma_z} = \mathrm{e}^{\pm i\omega t}\sigma_\pm \tag{3}$$

Problem 3

Instead of an electron, imagine now a proton in the same magnetic field. Show that in the co-rotating frame, $|\psi(t)\rangle = \mathrm{e}^{i\omega t\sigma_z/2}|\chi(t)\rangle$, the Schrodinger equation yields

$$\frac{\partial}{\partial t}|\chi(t)\rangle = \frac{i}{2}((\omega - \omega_0)\sigma_z + \gamma B\sigma_x)|\chi(t)\rangle \ . \tag{4}$$

Problem 4

In practice, one makes a CNOT gate from a system of two spins by applying the following transformation on them,

$$\mathrm{CNOT} \equiv \mathrm{e}^{-i\frac{\pi}{4}\sigma_y^1}\mathrm{e}^{i\frac{\pi}{4}\sigma_z^1\otimes\sigma_z^2}\exp^{i\frac{\pi}{4}\sigma_x^1} \ , \tag{5}$$

where the superscript in the Pauli matrix denotes the spin the matrix is acting on. Show that this transformation is equivalent to a CNOT operation (up to some phase factors).