2.111J/18.435J/ESD.79 Quantum Computation	Due: Nov 29, in class

Problem Set 8

Problem 1

Verify the following:

1. $[a, a^\dagger] = 1$
2. $[a^\dagger, a^\dagger a] = -a^\dagger$
3. $[a, a^\dagger a] = a$
4. $H = \frac{\hbar\omega}{2}(aa^\dagger + a^\dagger a) = \hbar\omega(a^\dagger a + \frac{1}{2})$

Problem 2

Define $|0\rangle = \int_{-\infty}^{\infty} \mathrm{e}^{-\frac{x^2}{2}} |x\rangle \, \mathrm{dx}$ (not normalized) and $|n\rangle \equiv (a^\dagger)^n |0\rangle$ (not normalized). Verify that

1. $a|0\rangle = 0$, where $|0\rangle$ is an eigenstate of H with eigenvalue $\frac{\hbar\omega}{2}$.
2. $a|n\rangle = \sqrt{n}\,|n-1\rangle$.
3. $a^\dagger |n\rangle = \sqrt{n+1}\,|n+1\rangle$.

Problem 3

The Jaynes-Cummings model is described by a Hamiltonian of the form,

$$H = \hbar\omega(a^\dagger a + \frac{1}{2}) + \frac{\hbar\omega_0}{2}\sigma_z + \hbar k(a^\dagger\sigma_- + a\sigma_+) \ .$$

Let $C = \omega(a^\dagger a + \frac{1}{2}\sigma_z)$ and $D = k(a^\dagger\sigma_- + a\sigma_+) - \frac{\Delta\omega}{2}\sigma_z$, where $\Delta\omega \equiv \omega - \omega_0$. Show that $H = \hbar(C + D)$, and that $[C, D] = 0$. This means that H commutes with both C and D. In addition, show that $C|n, \pm 1\rangle = \omega(a^\dagger a + \frac{1}{2}\sigma_z)|n, \pm 1\rangle = \omega(n \pm \frac{1}{2})|n, \pm 1\rangle$.