1) The key here is that we know \( M \), and \( 1 \ll M \ll N \).

So, rather than applying the operations

\[ |0\rangle \rightarrow \frac{1}{N} \sum_{i=1}^{N} |i\rangle = |s\rangle, \]

followed by \( \mathbb{I} - 2 |x\rangle \langle x| \)

followed by \( \mathbb{I} - 2 |s\rangle \langle s| = U_s \)

then repeating \( U_s U_\omega \) \( O(\sqrt{N/M}) \) times,

apply

\[ |0\rangle \rightarrow |s\rangle \]

followed by

\[ \mathbb{I} - 2 \sum_{j=1}^{M} |x_j\rangle \langle x_j| = U_\omega \]

followed by \( U_s \), and repeat

\[ U_s U_\omega \quad O\left(\sqrt{\frac{N}{M}}\right) \text{ times} \]