

This transforms the state $|S\rangle$ into a state very close (as in the original algorithm) to the state $\frac{1}{\sqrt{M}} \sum_{i=1}^M |w_i\rangle = |\tilde{w}\rangle$, as can be seen from the following argument.

Look at the subspace spanned by $|S\rangle$ and $|\tilde{w}\rangle$.

$$= \frac{1}{\sqrt{M-m}} \sum_{\substack{x \neq w_i \\ \text{for some } i}} |x\rangle$$

This is preserved by U_S , $U_{\tilde{w}}$, and \uparrow

$U_S U_{\tilde{w}}$ is easily seen to be a rotation by $\theta = 4 \sqrt{\frac{M}{N}}$ (actually $4 \sqrt{\frac{M-m}{M}}$) in this space.

Accordingly, after $\approx \frac{\pi}{4} \sqrt{\frac{N}{M}}$ iterations, the initial state $|S\rangle \approx |\tilde{S}\rangle$ is mapped into $|\tilde{w}\rangle$.

Measurement of the state at this point will reveal one of the $|w_i\rangle$ such that $\sum |w_i\rangle = 1$.