

5.2

So if system 1 = $|0\rangle_1$, this multiplies $|0\rangle_2$ by $e^{i\pi/4}$, $|1\rangle_2$ by $e^{-i\pi/4}$,

while if system 1 = $|1\rangle_1$, this

flips system 2 about the $\frac{1}{\sqrt{2}}\hat{y} = \frac{1}{\sqrt{2}}\hat{x}$ axis.

That is to say, it is like a CNOT, but with extra phases thrown in, and a rotation about a funny axis.

In coordinates:

$$U = \begin{matrix} & 00 & 01 & 10 & 11 \\ \begin{matrix} 00 \\ 01 \\ 10 \\ 11 \end{matrix} & \begin{pmatrix} e^{i\pi/4} & 0 & 0 & 0 \\ 0 & e^{-i\pi/4} & 0 & 0 \\ 0 & 0 & 0 & -e^{-i\pi/4} \\ 0 & 0 & e^{i\pi/4} & 0 \end{pmatrix} \end{matrix}$$

Compared with

$$U_{\text{CNOT}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$