

4) As noted in class, the set of effective Hamiltonians that one can apply ('do') given the ability to apply a set $\{H_j\}$ is given by $\pm i [H_j, H_k]$, $\pm [[H_j, H_k], H_L]$, etc.

So let's start commuting and see what we get.

$$i) \quad [x, p] = i, \quad \text{so get } \pm \mathbb{1}$$

$$ii) \quad [x, H] = [x, p^2/2] = \frac{1}{2}(xp^2 - p^2x) \\ = \frac{1}{2} \left(\underbrace{(xp - px)}_i p + p \underbrace{(xp - px)}_i \right) \\ = iP$$

iii) Similarly $[p, H] = -iX$, so ii) + iii) give nothing new