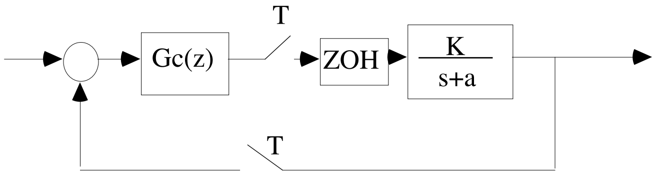
Design Example: Digital Control of a Velocity Servo:



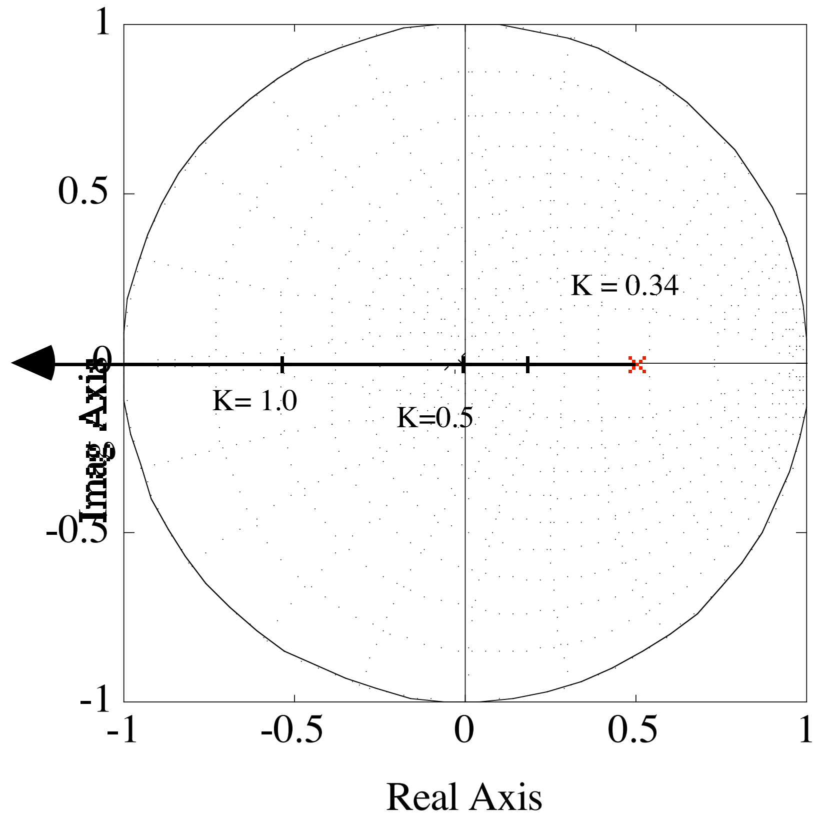
Assume a=1.4, and K=1, T = 0.5 sec

Plant equivalent = $(1-z^{-1})\dfrac{z(1-e^{-aT})}{(z-1)(z-e^{-aT})} = \dfrac{(1-e^{-aT})}{(z-e^{-aT})}$ which for the given values gives:

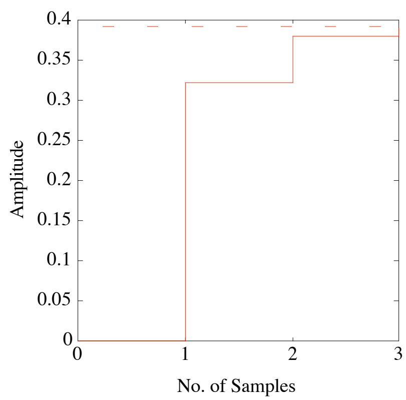
$$G_p(z) = \frac{0.5}{z-0.5}$$

Case #1 Simple Proportional Control

With $G_p(z) = K_p$, we get the following root locus

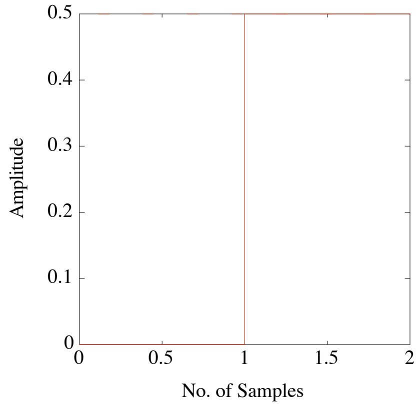


The resulting step responses for the root locations shown are:

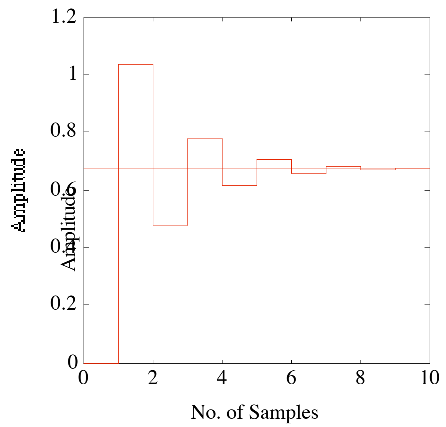


K=0.68 Note the slow but exponential response

Settling time = 3T Prediction = 4/ln(0.2) = 2.5



K=1.0 Chosen to be at the origin, giving a single time step response



K=2.0 On the negative real axis: Yield oscillations
Settling time = 7 T Prediction = 4/ln(0.55) = 7
Predicted damping = 0.2 therefore overshoot expected.

Note: Steady state error in each case is large, but does decrease with gain

Now consider adding an integrator to get ess = 0 . This implies adding a pole at the origin in s or at unity in z. We also will add a zero (at +0.2) to capture the roots. This gives us the forward loop transfer function:

$GcGp(z) = \frac{K(z-0.2)}{(z-0.5)(z-1)}$, which has the root locus:

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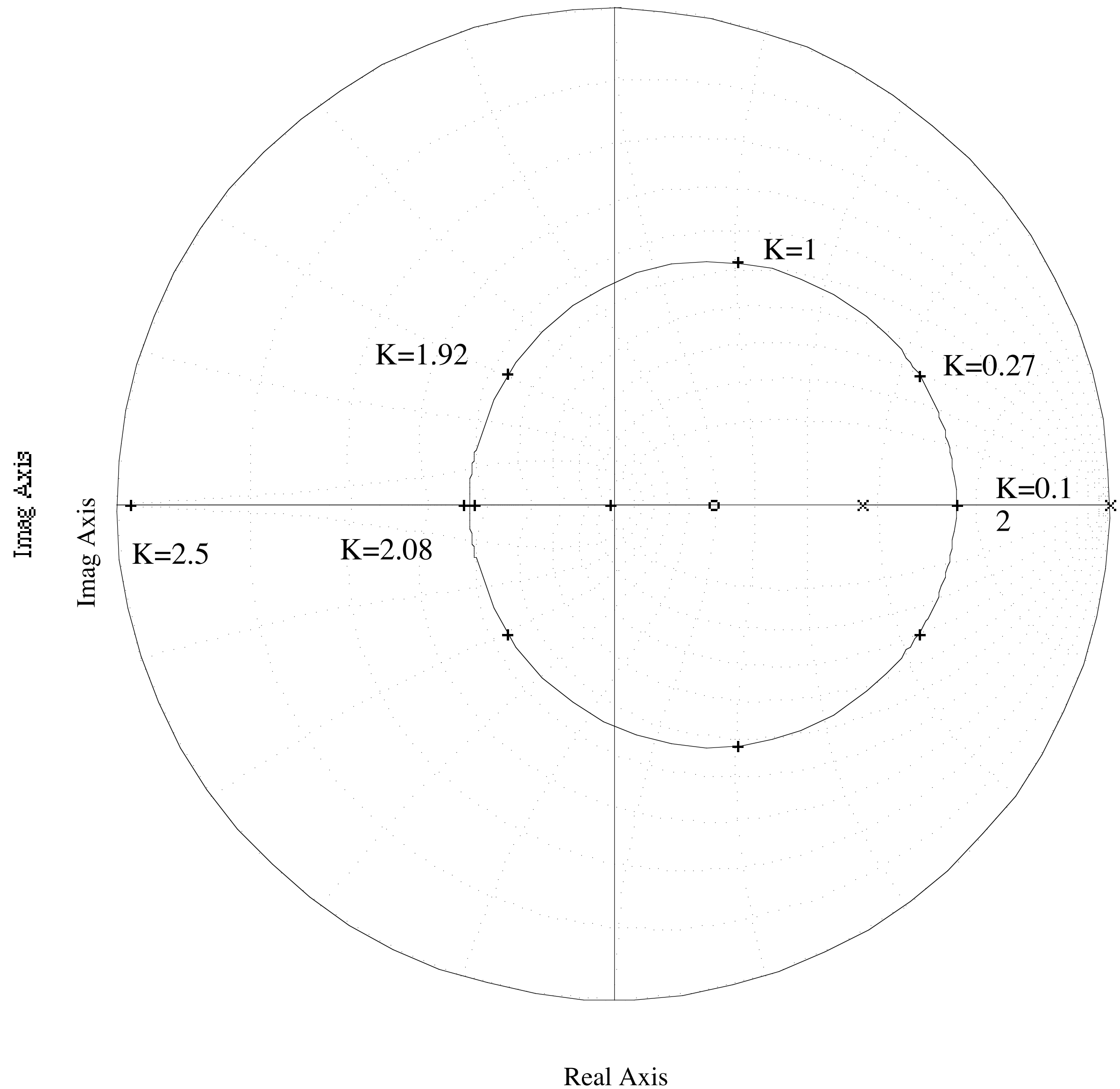
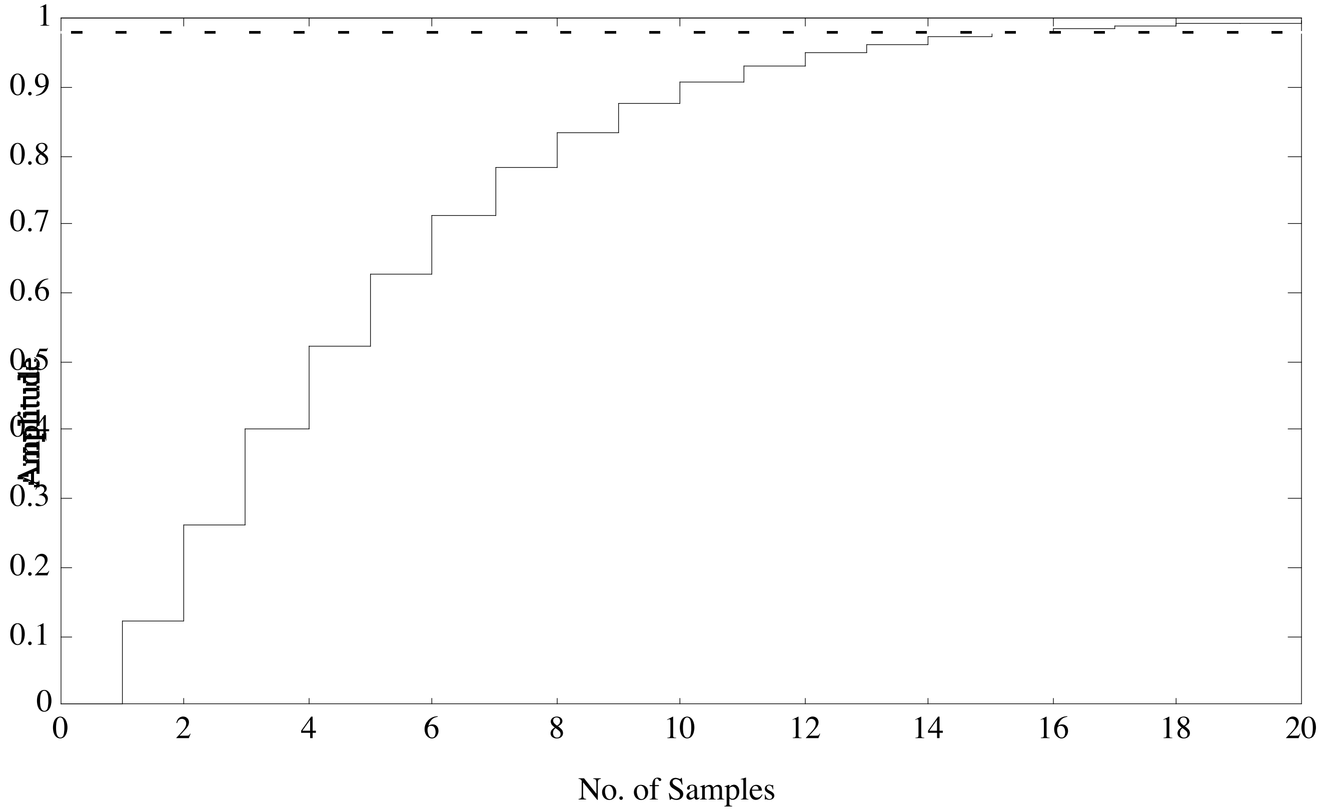
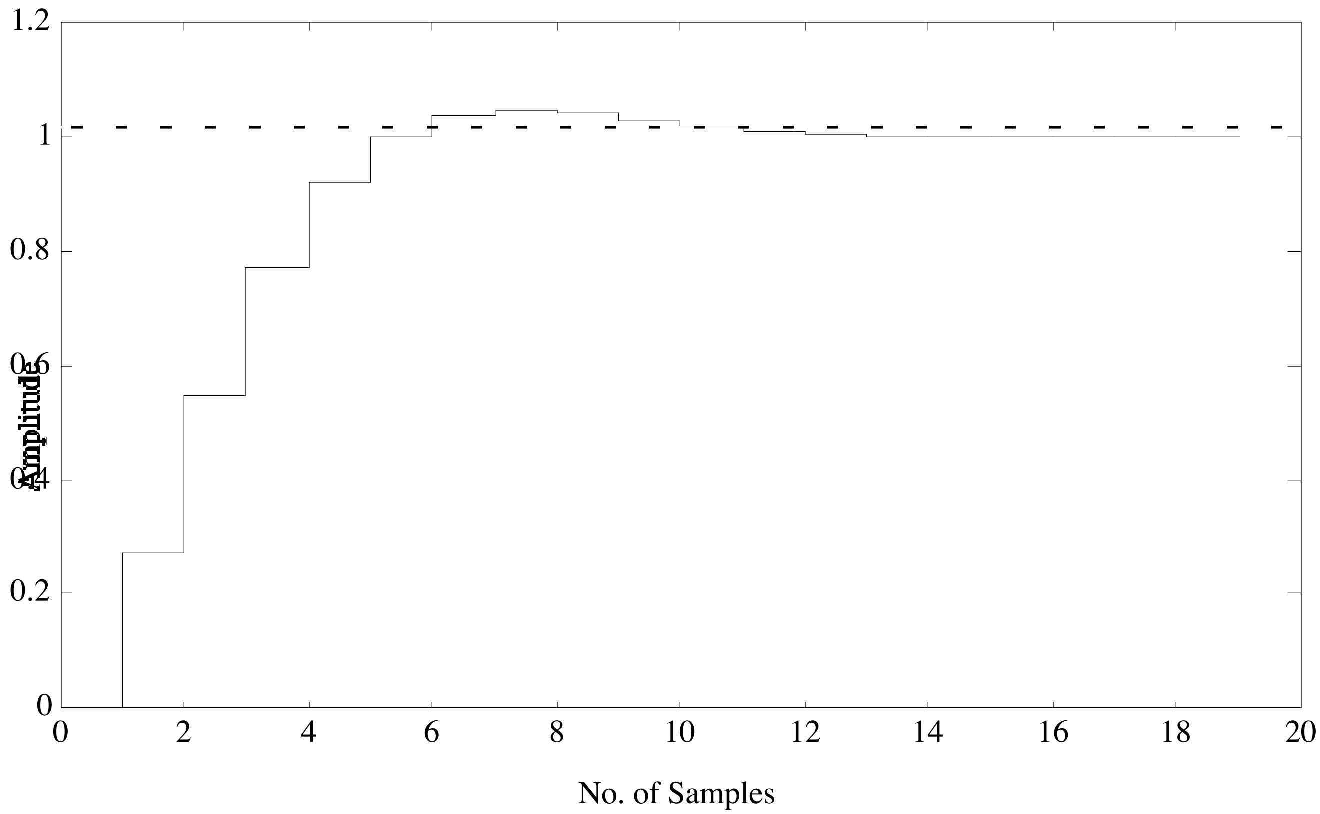




K = 0.12

Overshoot: None expected since roots are real; none occur in simulation

2% Setting Time: Since the distance from the origin ~ 0.7; we expect a setting time of $\frac{4}{\ln(0.7)}$ = 10.5 or 11(+1) time steps; the actual 2% settling time is indeed at time step 15, so we seem to underestimate this time..

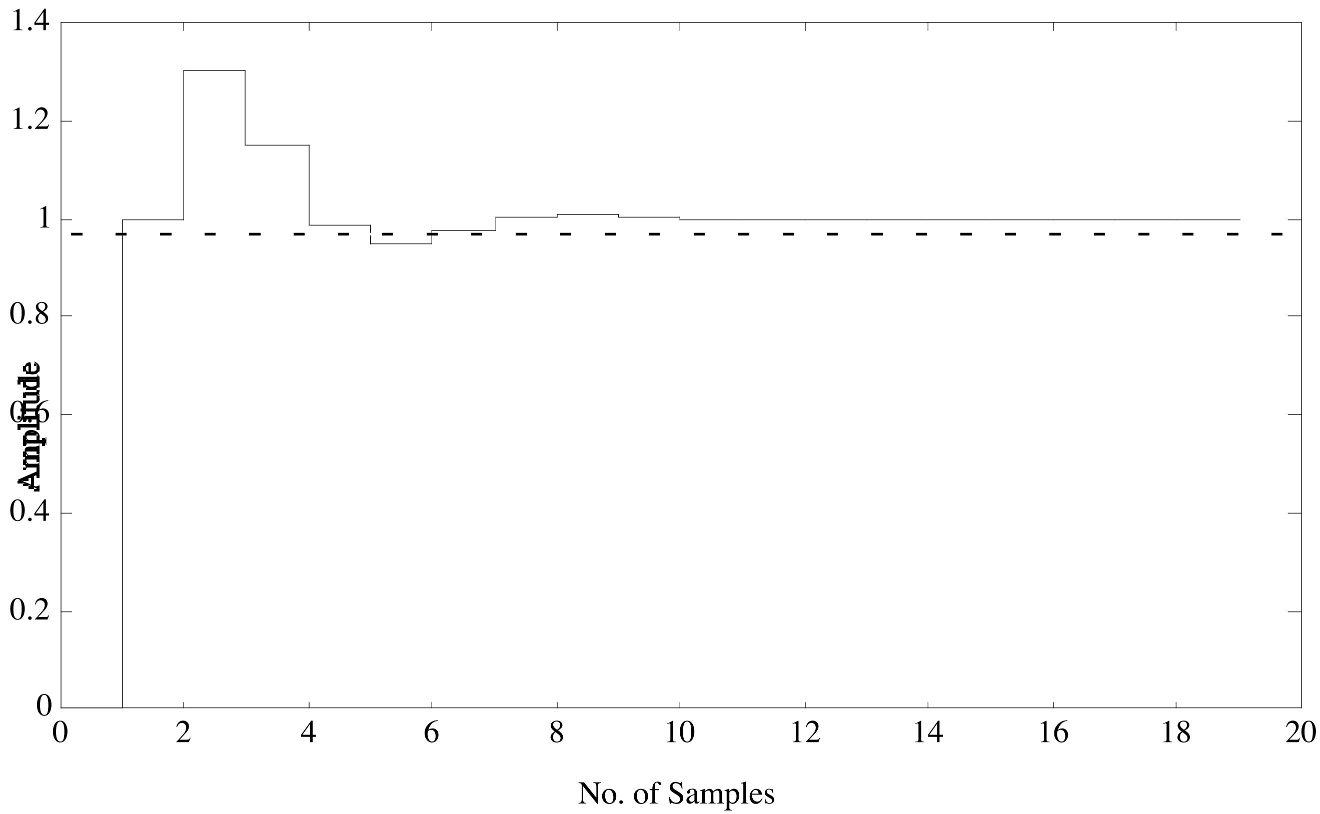


K= 0.27;

Observations:

Overshoot: The root locus indicates $\zeta \sim 0.7$ therefore we expect zero undershoot as is seen on the plot;

2% Setting Time: Since the distance from the origin ~ 0.66; we expect a setting time of $\frac{4}{\ln(0.66)} = 9.9$ or 10 time steps; the actual 2% settling time is indeed at time step 10.

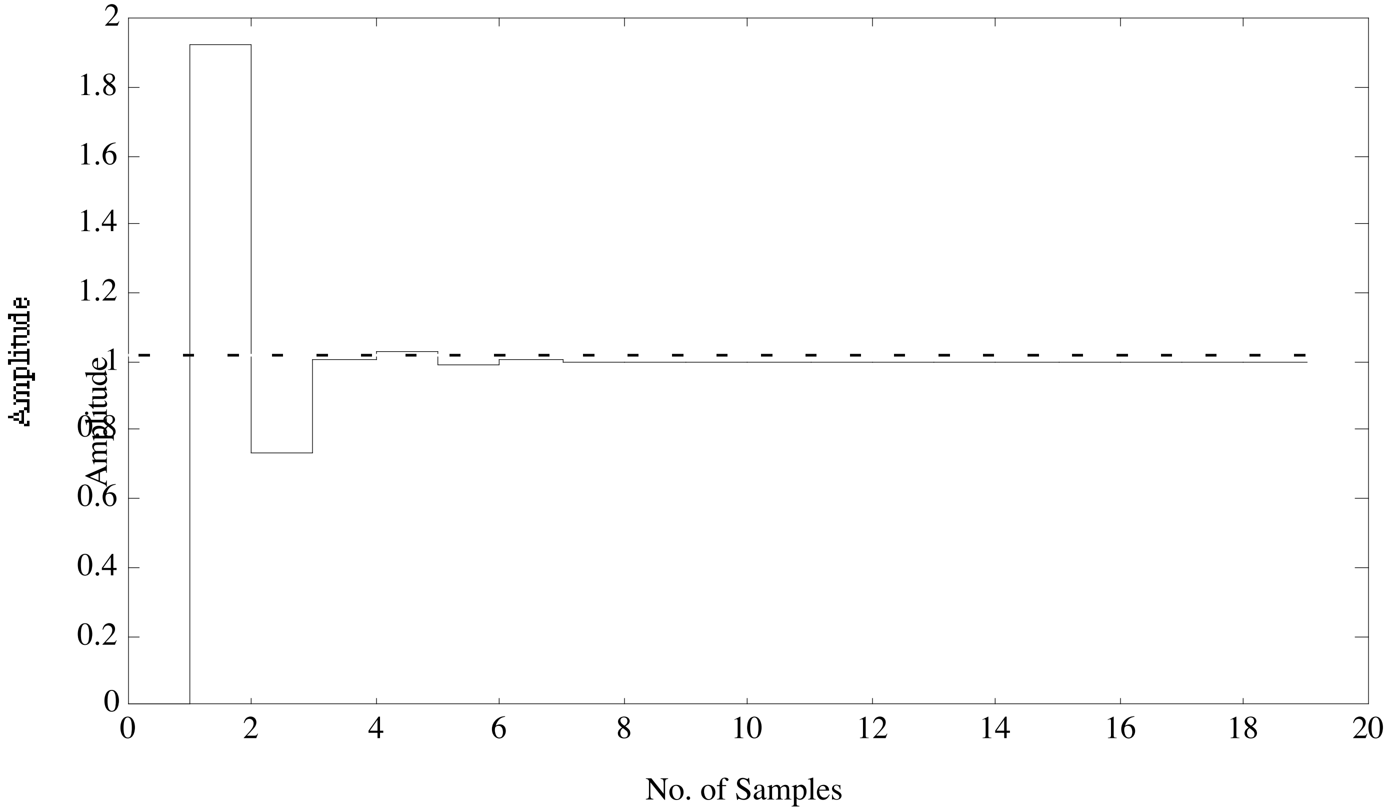


K= 1.0;

Observations:

Overshoot: The root locus indicates ζ <0.5 therefore we expect about 30% overshoot as is seen on the plot;

2% Setting Time: The distance from the origin has now decreased to about 0.33; we expect a setting time of $\frac{4}{\ln(0.33)}$ = 3.7 or 4(+1) time steps; the actual 2% settling time is at 6.

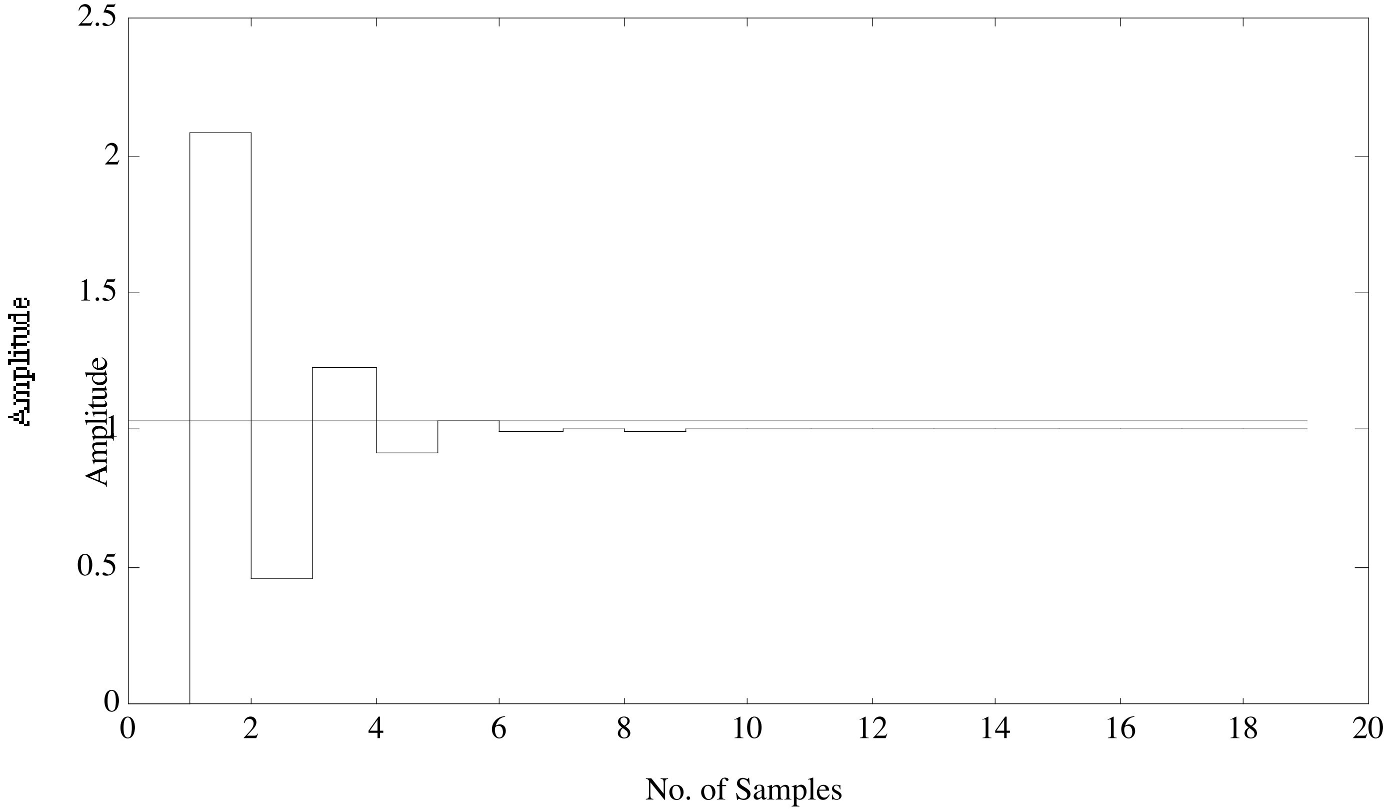


K= 1.92;

Observations:

Overshoot: The root locus indicates ζ ~4 therefore we expect about more overshoot; however the plot shows nearly 95% overshoot, which must in this case be caused by the closed-loop zero now having a greater effect.

2% Setting Time: The distance from the origin has now decreased to about 0.28; we expect a setting time of $\frac{4}{\ln(0.28)}$ = 3.1 or 4(+1) time steps; the actual 2% settling time is 5

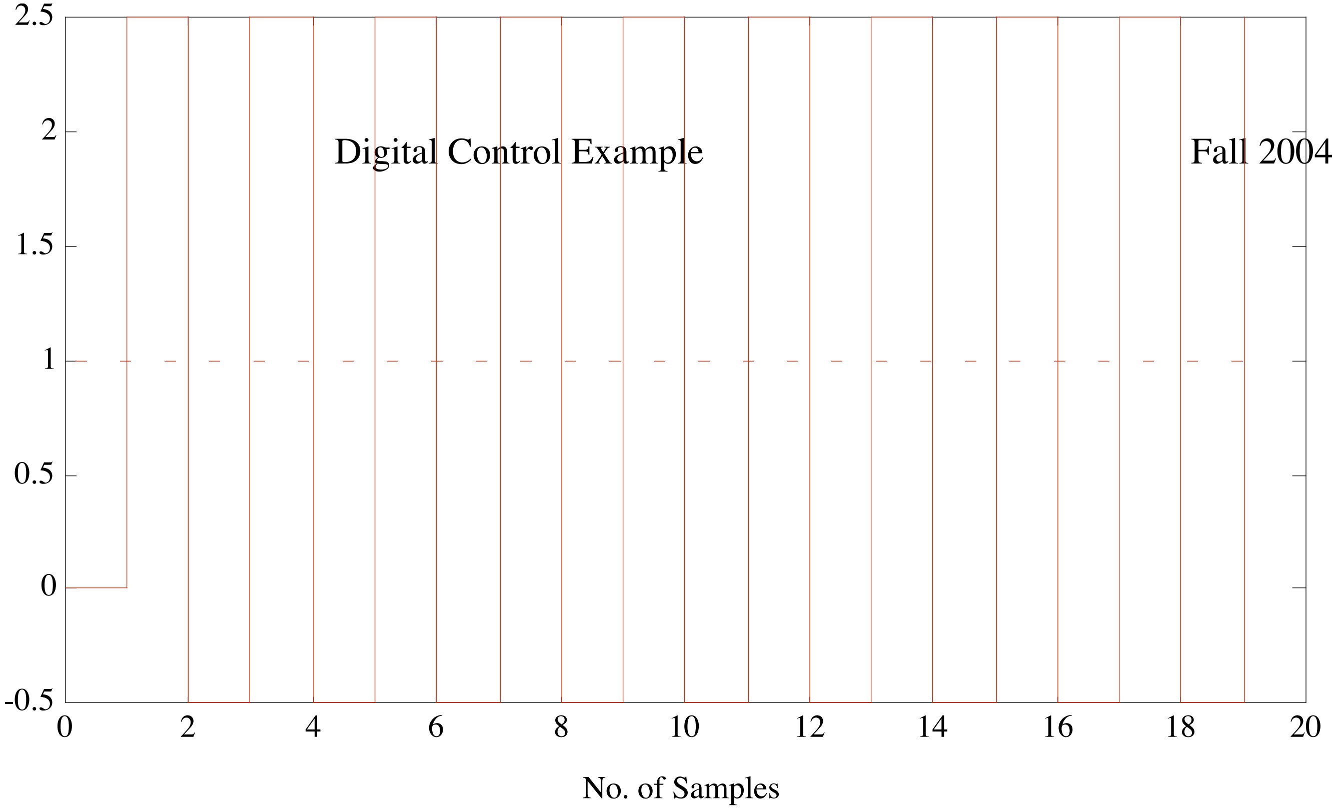


K= 2.08;

Observations: The roots are now real again but on the left half plane, thus $\zeta \neq 1$

Overshoot: The root locus indicates $\zeta \sim 0.4$ therefore we expect about 50% overshoot but again the overshoot is much greater; thus the zero is having an even greater effect.

2% Setting Time: The distance from the origin is still about 0.28; we expect a setting time of $\frac{4}{\ln(0.28)} = 3.7$ or 4(+1) time steps; the actual 2% settling time is at 5.



2.5
2
1.5
1
0.5
0
-0.5
0
2
4
6
8
10
12
14
16
18
20
Digital Control Example
Fall 2004
No. of Samples