Design Example: Digital Control of a Velocity Servo:

Assume $a=1.4$, and $K=1$, $T = 0.5$ sec

Plant equivalent $= \frac{z(1-e^{-aT})}{(z-1)(z-e^{-aT})} = \frac{(1-e^{-aT})}{(z-e^{-aT})}$ which for the given values gives:

$G_p(z) = \frac{0.5}{z-0.5}$

Case #1  Simple Proportional Control

With $G_p(z) = K_p$, we get the following root locus
The resulting step responses for the root locations shown are:

K=0.68  Note the slow but exponential response
Settling time = 3T  Prediction = $\frac{4}{\ln(0.2)} = 2.5$

K=1.0  Chosen to be at the origin, giving a single time step response

K=2.0  On the negative real axis: Yield oscillations  
Settling time = $7 \bar{T}$ Prediction = $\frac{4}{\ln(0.55)} = 7$  
Predicted damping = 0.2 therefore overshoot expected.

Note: Steady state error in each case is large, but does decrease with gain
Now consider adding an integrator to get \( \text{ess} = 0 \). This implies adding a pole at the origin in \( s \) or at unity in \( z \). We also will add a zero (at +0.2) to capture the roots. This gives us the forward loop transfer function:

\[
G_cG_p(z) = \frac{K(z-0.2)}{(z-0.5)(z-1)} , \text{ which has the root locus:}
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This gives us the forward loop transfer function:

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which has the root locus:
K = 0.12

Overshoot: None expected since roots are real; none occur in simulation

2% Setting Time: Since the distance from the origin ~ 0.7; we expect a setting time of $\frac{4}{\ln(0.7)} = 10.5$ or $11(+1)$ time steps; the actual 2% settling time is indeed at time step 15, so we seem to underestimate this time.
K = 0.27;

Observations:

**Overshoot:** The root locus indicates $\zeta \sim 0.7$ therefore we expect zero undershoot as is seen on the plot;

**2% Setting Time:** Since the distance from the origin $\sim 0.66$; we expect a setting time of $\frac{4}{\ln(0.66)} = 9.9$ or 10 time steps; the actual 2% settling time is indeed at time step 10.
K = 1.0;

Observations:

**Overshoot:** The root locus indicates $\zeta < 0.5$ therefore we expect about 30% overshoot as is seen on the plot;

**2% Setting Time:** The distance from the origin has now decreased to about 0.33; we expect a setting time of $\frac{4}{\ln(0.33)} = 3.7$ or $4(+1)$ time steps; the actual 2% settling time is at 6.
K = 1.92;

Observations:

**Overshoot:** The root locus indicates $\zeta \sim 4$ therefore we expect about more overshoot; however the plot shows nearly 95% overshoot, which must in this case be caused by the closed-loop zero now having a greater effect.

**2% Setting Time:** The distance from the origin has now decreased to about 0.28; we expect a setting time of $\frac{4}{\ln(0.28)} = 3.1$ or $4(+1)$ time steps; the actual 2% settling time is 5
K = 2.08;

Observations: The roots are now real again but on the left half plane, thus $\zeta \neq 1$

**Overshoot:** The root locus indicates $\zeta \sim 0.4$ therefore we expect about 50% overshoot but again the overshoot is much greater; thus the zero is having an even greater effect.

**2% Setting Time:** The distance from the origin is still about 0.28; we expect a setting time of $\frac{4}{\ln(0.28)} = 3.7$ or $4(+1)$ time steps; the actual 2% settling time is at 5.