

# Department of Mechanical Engineering

## 2.010 CONTROL SYSTEMS PRINCIPLES

### Laboratory 2: Characterization of the Electro-Mechanical Plant

**Introduction:** It is important (for future lab sessions) that we have an adequate model, and numerical values for the parameters of the electro-mechanical motor/load plant at your lab station. In this session your goal is make measurements on the various components of the system and to determine the parameters. The specific objectives are:

- To measure the gain  $K_a$  of the servo-amplifier.
- To measure the motor torque constant  $K_m$ .
- To estimate and measure the shaft/load moment of inertia  $J$ .
- To investigate and characterize the rotational friction characteristics of the motor shaft and bearings.
- To measure the tachometer gain constant  $K_t$ .

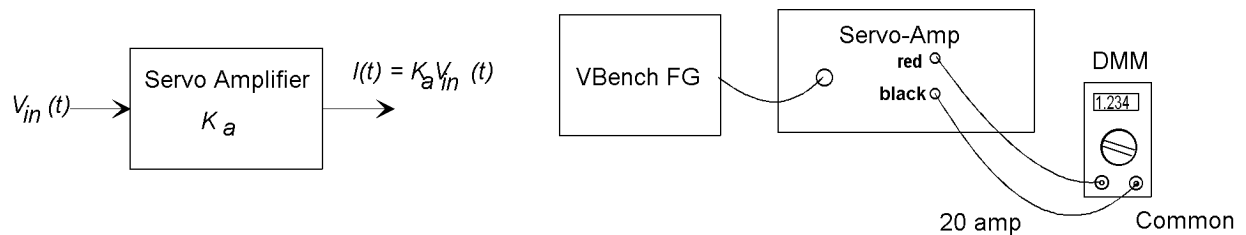
**Preparation:** Review the attached specification sheet for the Aerotech 1135-01 servo motor. Also you can save time in the lab by sketching the step responses asked for in part (a) of the Mechanical Properties section below - “Is the shaft friction viscous?”

### Measurements:

**Amplifier Gain Constant  $K_a$ :** The servo-amplifier is a *voltage-controlled current-source*, that is it generates an output *current* that is proportional to the *voltage* that is applied at the BNC input connector. The amplifier will adjust its output voltage to maintain the output current (up to its saturation limits of  $\pm 40$  volts). Within the linear range the input/output relationship is

$$I_{out}(t) = K_a V_{in}(t)$$

The task is to measure the gain constant  $K_a$ .



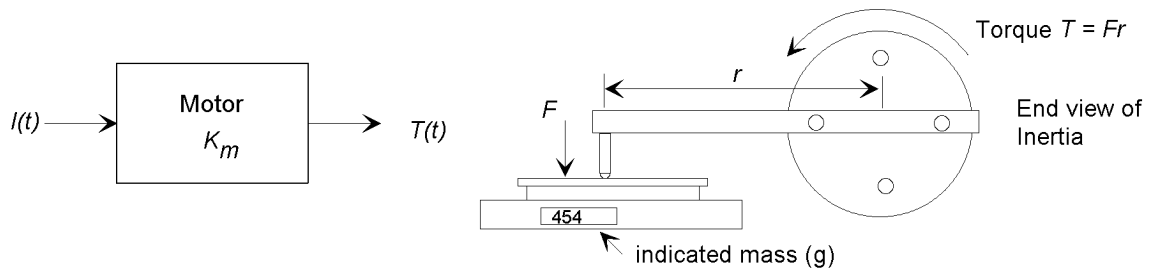
**Procedure:** Use the Virtual Bench Function Generator to generate a steady (dc) voltage by using the *Offset* control. Connect the D/A output to the servo-amplifier input. Make sure the amplifier is turned off and disconnect the servo-motor from the red and black terminals. Connect the Wavetek DMM using its 20 amp current input terminal. Set the meter to the AMP range and press the yellow button to select dc measurements. Note: Be careful making these connections – you can damage the DVM and/or the amplifier through incorrect connections.

The nominal amplifier gain is  $K_a = 2 \text{ A/V}$ , and the maximum sustained current that the amplifier can supply is 5 A. Select an appropriate input voltage and turn on the function generator. Measure the current and calculate the gain constant in the space below.

**The Motor Torque Constant** For a permanent-magnet dc motor the torque produced is proportional to the armature current:

$$T(t) = K_m I(t)$$

The task is to measure the torque constant  $K_m$ . (Note that the Aerotech spec sheet calls this quantity  $K_t$ .)



**Procedure** Reconnect the motor to the servo-amp. You are given a digital postage scale and a lever arm that clamps on to the aluminum inertial load. The function generator may be used to generate a given motor current, and the indicated mass shown on the scale may be used to calculate the force applied by the lever arm, and hence the torque.

**Note:** In measurements such as this it is common to apply a “dither” signal to overcome any “stiction” in the bearings or the scale. We suggest that you set the function generator to add a 0.25 volt sinusoid at approximately 30 Hz to the dc value you use in order to minimize stiction effects.

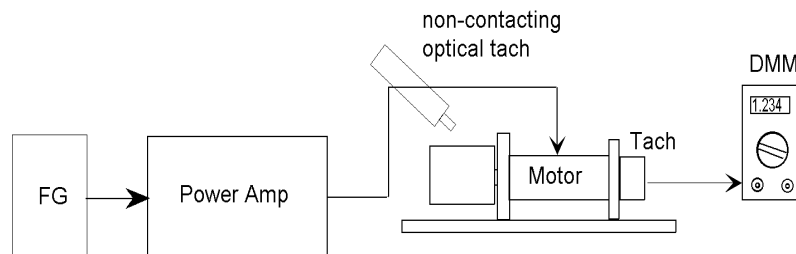
Use the function generator to measure the indicated mass at output voltages of 0, -1, -2, -3, -4 volts. **Note:** Be very careful about the polarity of the voltage – a positive voltage will cause the motor to rotate in the wrong direction.

On your Windows 95 desktop you will find a folder named *2.010 Files*. Open the folder and then open the *Lab 2* folder. Open the Excel file *Motor Calibration* and enter your data into the spreadsheet, which will plot the torque/current relationship and fit a straight line to it. The slope of the line is the torque constant  $K_m$ . Print the spreadsheet and graph on a single page (attach it) and enter the result below. Compare your measured value with the manufacturer’s quoted value from the data sheet.

**The Tachometer Gain Constant** The tachometer serves as the angular velocity sensor for closed-loop control. It is an electromagnetic transducer that generates a voltage that is directly proportional to the angular velocity  $\Omega$  of the shaft.

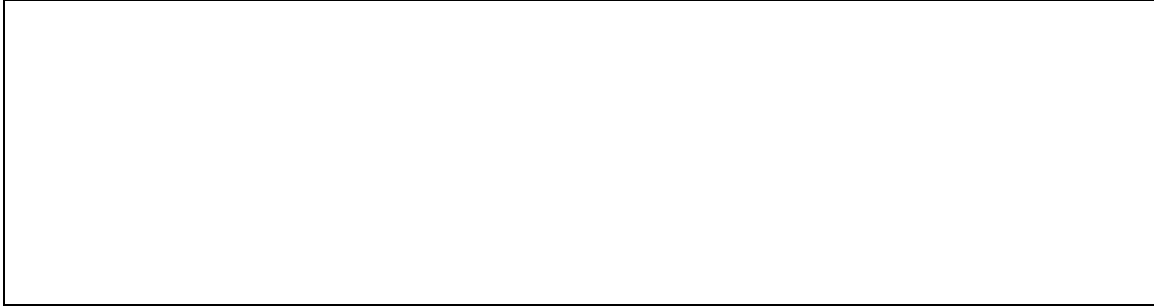
$$v_t = K_t \Omega(t)$$

The task is to measure the constant  $K_t$  for your motor.



**Procedure:** With the setup as before, remove the torque monitoring lever arm and connect your DMM to the tachometer BNC connector on the motor terminal box. Apply a small dc voltage using the function generator and let the motor spin. The lab instructor will help you to measure the angular velocity with a non-contacting optical

tachometer. Record both the measured speed and the tachometer voltage and compute the gain constant  $K_t$ . Compare your measured value with the manufacturer's quoted value from the data sheet.



**Motor Mechanical Properties (Shaft Inertia and Friction)** Our analyses of motor behavior have assumed a linear lumped parameter model of the mechanical system (moment of inertia  $J$  and viscous friction  $B$ ). We now test the assumption of viscous friction and make measurements of the parameters.

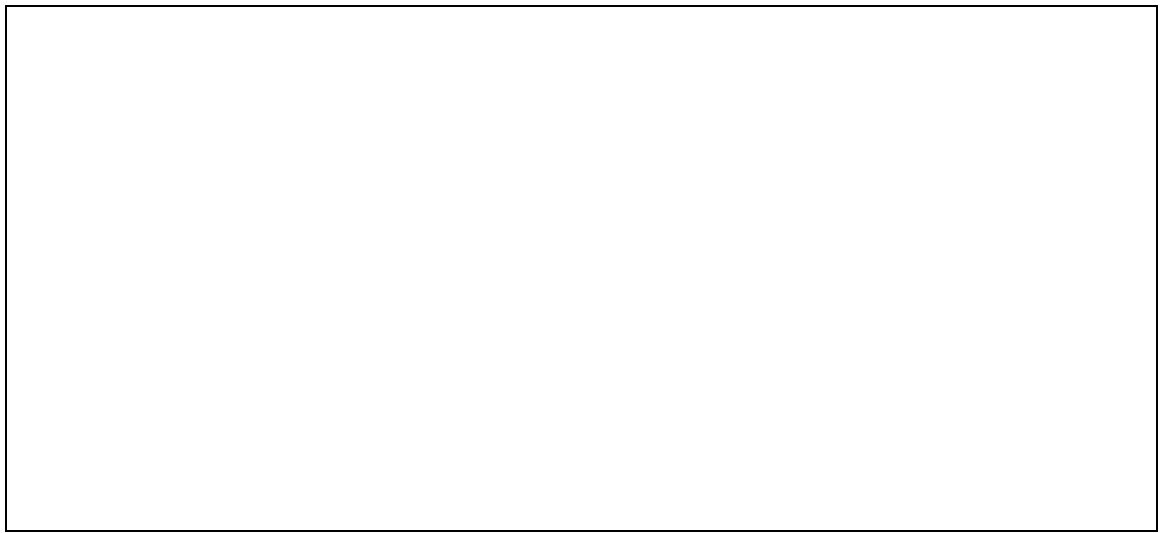
- (a) **Is the shaft friction viscous?** Consider two possibilities, (i) the frictional torque  $T_f$  is viscous in nature, that is  $T_f = B\Omega$ , in which case the differential equation describing the angular velocity is:

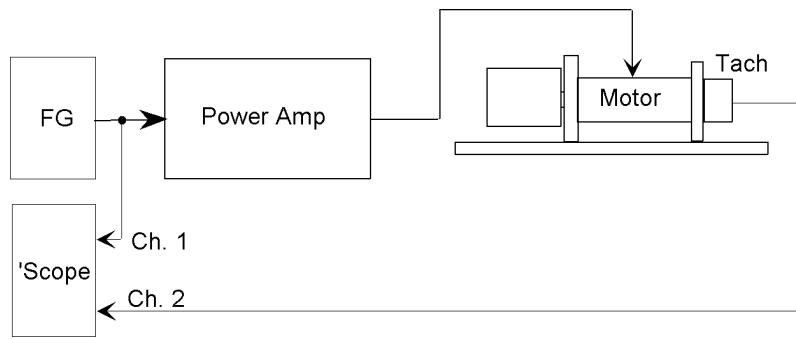
$$J\frac{d\Omega}{dt} + B\Omega = T(t) \quad (1)$$

where  $J$  is the shaft moment of inertia,  $B$  is the viscous coefficient, and  $T$  is the motor torque, or (ii) the friction is Coulomb (or static) in nature, that is  $T_f = \text{sgn}(\Omega)K_f$  and depends only on the sign of the shaft velocity. (The signum function  $\text{sgn}()$  ensures that the the frictional torque opposes the motion.) In this case

$$J\frac{d\Omega}{dt} = T(t) - T_f \quad (2)$$

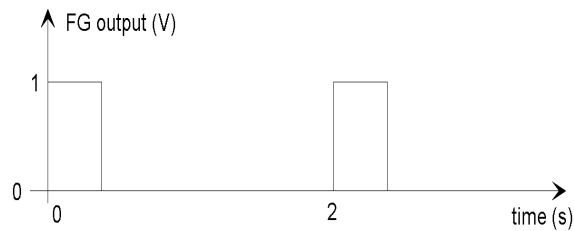
In the space below make sketches of the *form* of the angular velocity response you would expect from a step in the applied torque  $T$  for each of the two cases.





**Procedure:** Set up the function generator to generate a 10% duty cycle, 1 volt unipolar square wave with a 2 second period as follows:

Frequency	0.5 Hz
Duty Cycle	10 – 15%
Amplitude	0.5 V
Offset	0.5 V



Use the square wave to drive the motor, and record the square wave on Ch. 1 of the oscilloscope, and the angular velocity  $\Omega$  from the tachometer on Ch. 2 of the scope. Adjust the duty cycle to ensure that the motor comes to rest between pulses. Which model of the friction best describes the observed velocity response? Write your answer with reasons below:

- (b) **Determine the Moment of Inertia:** Assume from this point that the shaft friction is purely Coulomb in nature, that is the system is described by Eq. (2). Our task is to estimate  $J$  and  $T_f$  from the measured angular velocity response.

Consider the two regions of the response. In the first the generator voltage is 1 volt, the shaft is accelerating and the equation of motion is

$$\dot{\Omega}_1 = \frac{d\Omega}{dt} = \frac{1}{J} (K_a K_m V - T_f) \quad (3)$$

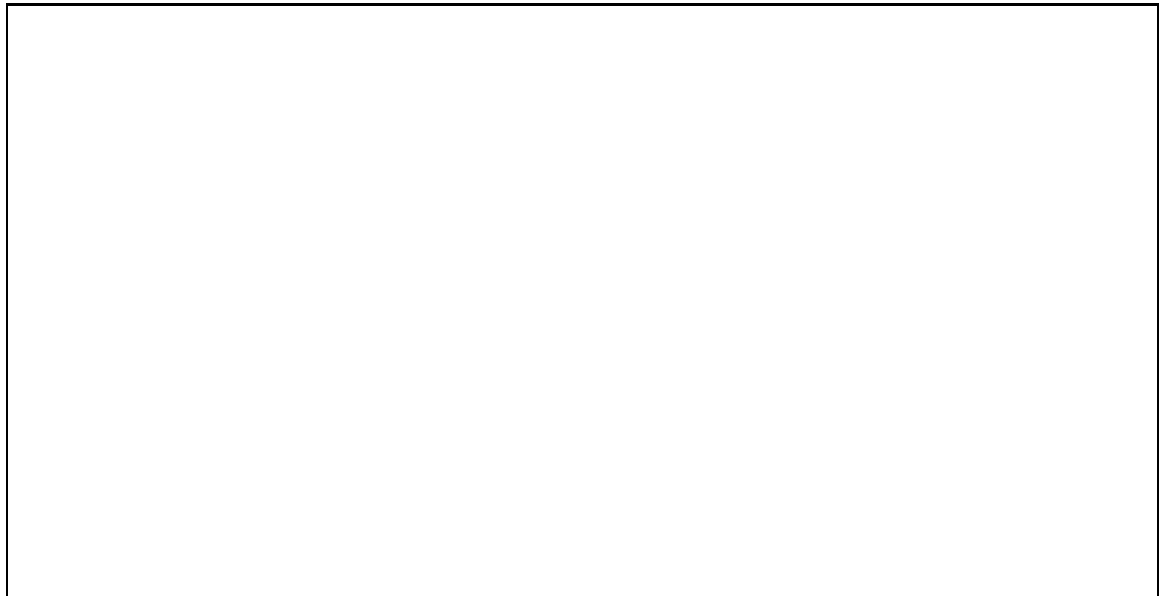
where  $V$  is the voltage (1 volt). In the second (“coast-down”) region where the voltage is zero there is no applied torque and the equation becomes

$$\dot{\Omega}_2 = \frac{d\Omega}{dt} = -\frac{1}{J} T_f \quad (4)$$

Eqs. (3) and (4) may be combined to solve for  $J$

$$J = \frac{K_a K_m V}{\dot{\Omega}_1 - \dot{\Omega}_2} \quad (5)$$

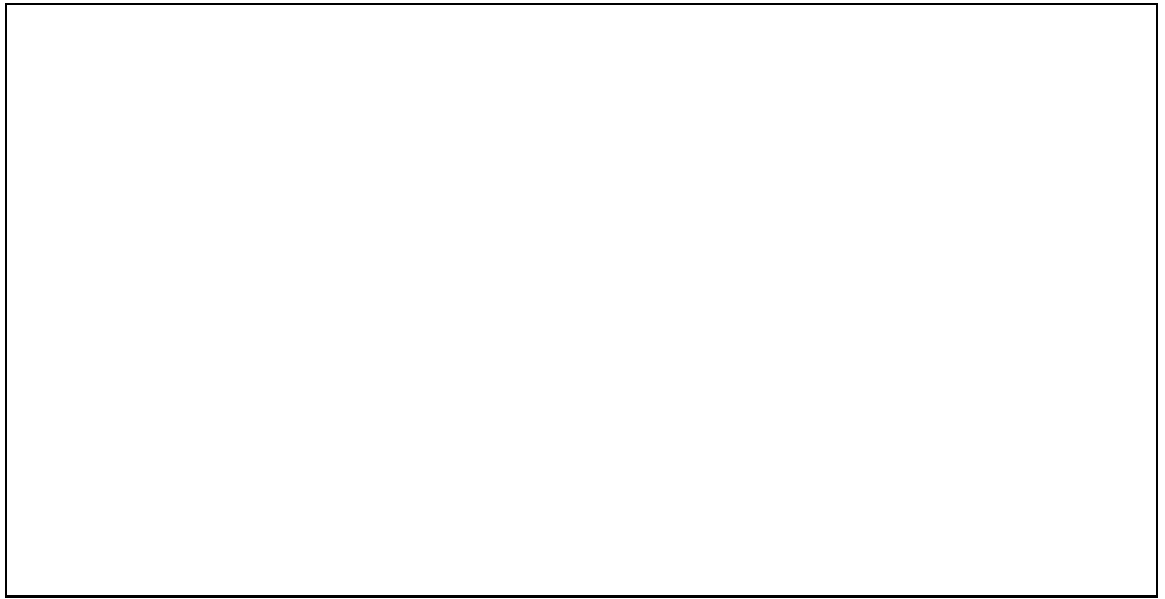
Use the oscilloscope to measure the slopes of the two regions of the response. (We suggest you use the cursors and the  $dV$  and  $dt$  measurements). Calculate the value of  $J$  below:



To verify your estimate of  $J$ , calculate the value from the dimensions of the aluminum wheel and the motor shaft as follows: The diameter of the wheel is 76 mm., its length is 76 mm, and the density of aluminum is 2600 kg/m<sup>3</sup>. The moment of inertia of a disk is  $J_{disk} = mr^2/2$ . The total inertia is

$$J = J_{disk} + J_{shaft}$$

where  $J_{shaft}$  is specified in the attached spec sheet. Compute the shaft inertia in the space below:



- (c) **Determine the Coulomb Friction Torque:** Use Eq. (2) to estimate the friction torque  $T_f$  from your measurements. Compare the value with that given in the spec sheet.





## Summary:

Summarize your measured values of the electro-mechanical plant parameters below (with units):

Amplifier Gain Constant ( $K_a$ ):

Motor Torque Constant ( $K_m$ ):

Load Inertia ( $J$ ):

Coulomb Friction Torque ( $T_f$ ):

Tachometer Gain Constant ( $K_t$ ):