Department of Mechanical Engineering

2.14 ANALYSIS AND DESIGN OF FEEDBACK CONTROL SYSTEMS

Laboratory 6: Frequency Response Measurement

Laboratory Objectives:

(i) The measurement of the closed-loop frequency response characteristics of the laboratory motor/load system with the control system you designed in Laboratory 5.

(ii) The construction of Bode plots.

(iii) An introduction to frequency response plots in polar form.

(iv) The comparison of the experimentally determined frequency response with a predictions from the system model.

Preparation

Review material from 2.003 and 2.004 on frequency response and Bode plots.

Introduction

A thorough understanding of frequency response characteristics is essential for control system design. In this laboratory we ask you to measure the sinusoidal response of a closed-loop system, and to plot the response in two common forms: the Bode plots, and the polar plot.

When a sinusoidal input $u(t) = A\sin(\omega t)$ excites a linear system, the output is a sinusoid with the same frequency, but with a modified amplitude and phase, defined by the frequency response function $H(j\omega)$

$$y(t) = A\,|H(j\omega)|\sin\left(\omega t + \angle H(j\omega)\right)$$

The magnitude $|H(j\omega)|$ may be determined by measuring the *ratio* of the output amplitude $AH(j\omega)$ to the input amplitude at the given frequency ω, while the phase $\angle H(j\omega)$ between input and output can be estimated from the temporal delay between input and output.

Procedure (See the hints below)

Perform a detailed measurement of the the frequency response of the closed-loop system that you designed in Laboratory 5 as follows:

(1) Set up the position control system you designed for Laboratory 5, driven by the function generator, and monitor the function generator output and the position response. Use sinusoidal excitation.

(2) Make a measurement of the frequency response, recording input frequency and amplitude, output amplitude, and the time by which the output sine wave leads or lags the input (remember to record time lags with an overall minus sign).

(3) When you have enough points to map out the frequency response accurately, go to the 2.010 folder and open the Excel worksheet Lab6. This has been prepared for you to enter the data and automatically generate frequency response plots in Bode, polar and log magnitude-phase form. Record your data points in the appropriate columns of the worksheet. The remaining (shaded) columns will be calculated automatically and the plots will be found on "sheets" designated on the tabs at the lower left of the spreadsheet. (Record your results in ascending order of frequency; alternatively, you can record them in any order and then perform a sort (on the whole input block). If the results are not sorted by frequency, they will not plot properly.) You may need to iconify the spreadsheet to collect more data points in interesting regions. When you have collected enough points, print out the Bode magnitude/phase and polar plots.

(4) From the Bode magnitude/phase plots estimate the total number and location of poles and zeroes. If there are pole pairs, estimate the damping ratio. (For a lightly damped second-order system the magnitude of the resonant peak is approximately $1/(2\zeta)$ times the low frequency response mgnitude.) Record your results. Be sure to estimate your experimental error.

(5) Use the `bode` function in Matlab to compute the frequency response for your closed-loop system. Compare your measured plots with that predicted from Matlab. Do they agree? If not, why not?

<u>Hints:</u>

(i) Since you will be plotting your results as a function of frequency on a logarithmic scale, it makes sense to start out by making measurements at 1 Hz, 2 Hz, 4 Hz, 8 Hz, 16 Hz, etc., in order to cover the frequency range evenly.

(ii) Once you have mapped out the general features of the frequency response, return and make more detailed measurements in regions that look interesting (for example, in the neighborhood of resonances, rapid phase shifts, etc.). As a rule, concentrate measurement points in regions of high curvature.

(iii) Since in our controllers output and input are inverted with respect to each other, use the Channel Setting in the Virtual Bench oscilloscope to proved a gain of -2 to the tachometer output (recall that the factor of 2 is to compensate for the voltage divider in the input/output box).

(iv) To measure amplitudes and time delays efficiently, put one scope cursor on the input and the other on the output and use the cursors to measure amplitudes and time delays automatically. (Remember that the cursors measure amplitudes from the center of the screen, so make sure your traces are centered vertically.)

(v) Be sure to distinguish between leading and lagging phase.

(vi) You will need to use smaller input amplitudes at some frequencies to prevent blowing the circuit breaker. You can the oscilloscope's averaging feature to provide cleaner signals at high frequencies.