2.14 Problem Set 1 Solutions

Problem 1

Pressure regulator on gas grill:
a) knob setting should correspond to some particular, even pressure flow of gas, resulting in even cooking flame.
b) actual flame quality (eveness, etc); cooking temp (flame size, etc)
c) gas pressure
d) knob position/angle
e) hose fluid dynamic system and valve/spring mechanism
f) force/pressure exerted by gas stream wrt atm.
g) ambient air pressure (due to elevation; weather); leaks

Water closet (toilet tank):
a) want stored volume of water, appropriate to obtain good flush
b) consistent volume kept between flushes
c) water volume (water level)
d) ref. input is height where float-lever shuts off water in-flow
e) float attach to lever, which toggle valve for water in-flow; overflow tube that allows level to go down if height goes above ref.
f) height of water in tank
g) leaks; objects in tank (empty plastic bottle) that displace water

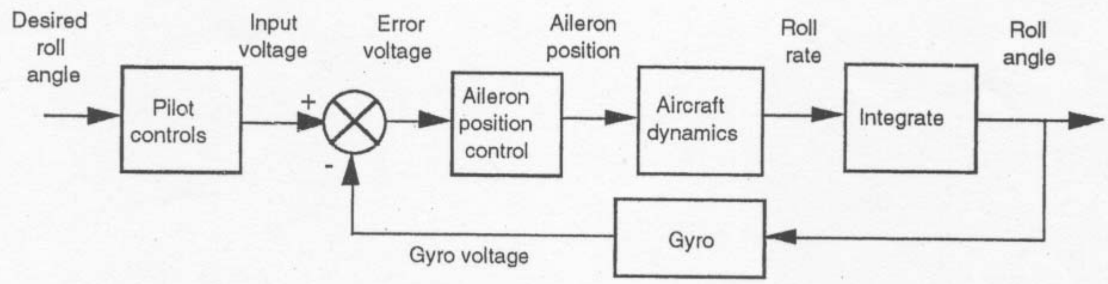
Shower mixing valve:
a) want knob setting to correspond to particular temp water from shower
b) constant temperature (and flow rate) water desired
c) output variable is mixed water output temperature
d) ref. input is knob angle
e) valve responds to temperature and pressure changes in each of the input (hot and cold) water flows, metal or wax expands or contracts with temperature changes to adjust each flow rate and approach desired set point at output
f) temperature of mixed water
g) temperature and pressure variations in the hot and cold water feeding into the valve (due to toilet flushes, city supply line pressure drop, etc)

Iris of your eye:
a) goal to regulate intensity of light entering eye
b) apparent "brightness" of images perceived reasonable; no damage to eye
c) amount of light entering eye
d) good light level set by eye ("hardware") and/or brain
e) iris opening, muscle tissue controlling diameter
f) light intensity
g) ambient and direct light to eye, eyelids blocking some of light,

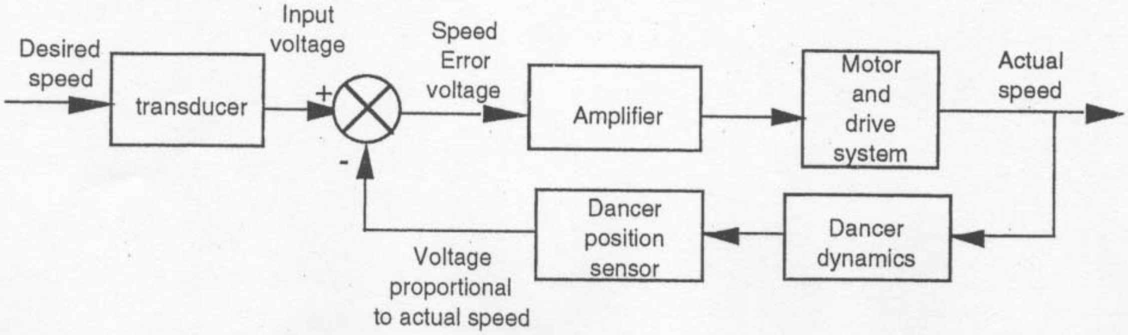
Angle of your elbow:
a) want to position arm to manipulate objects, etc
b) actual position and angle of arm; dexterity; fast response; no overshoot; etc
c) arm angle, steadiness
d) desired angle
e) eye gauges position, brain sends electrical signals to correct, muscles respond
f) visual (and "sense of body position") feedback to the brain
g) heavy objects being lifted; fatigue of muscles; being pushed, etc

Monetary inflation:
a) want to keep inflation rate small (between 0 and 4% annually?)
b) cost of goods and exchange rate with other nations does not become "expensive" over time
c) percentage change in cost of consumables; other currencies; Consumer Price Index
d) inflation rate
e) the Federal Reserve sets the lending rate (discount rate) for banks for short-term borrowing. Buying and selling govt securities (T-bills, etc). Indirectly stimulates (or dampens) the economy, by effectively making it more or less expensive to borrow money.
f) bank interest rates; consumer price index; exchange rates
g) economies of other nations; world politics (wars, etc, causing economic uncertainty); price of competing investment opportunities (gold, etc)

3. Problem 2, Nise 1.3



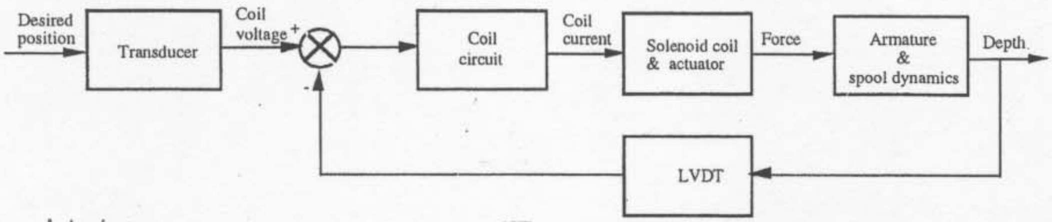
4. Problem 3, Nise 1.4



Problem 4, Nise 1.12

I give both a "less-detailed" and "more-detailed" answer... either is OK.

12.



More-detailed:

Note that a "voice coil" and a "solenoid" are both linear systems which operate under the same physics studied in the familiar MOTOR models ("voltage-controlled" and "current-controlled") from 2.003, except we have:

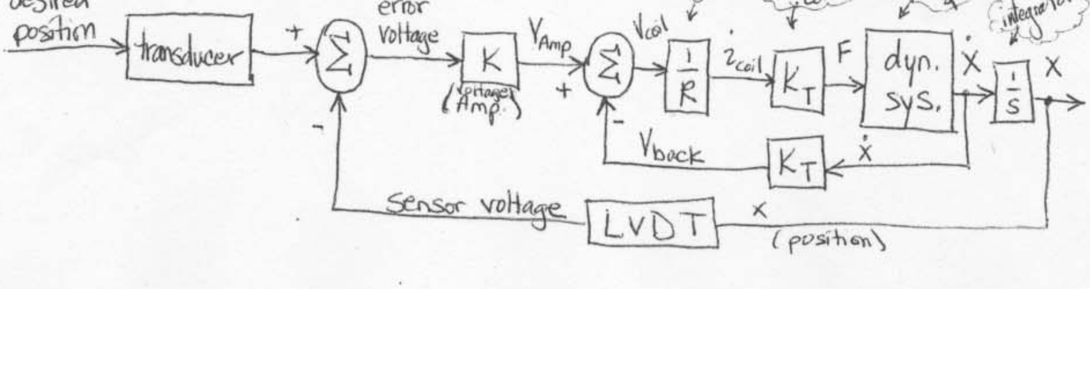
v.c. or solenoid { force and linear velocity } (instead of) { torque and angular velocity } motor

Since we are controlling voltage instead to commanding current directly, there is a feedback loop involving the back emf [a voltage proportional to the velocity of the coil in the magnetic field] which is subtracted from the commanded voltage. Here are some relationships:

$\boxed{F = K_T i_{coil}}$ either $\boxed{i_{coil} = \frac{V_{coil}}{R_{coil}}}$ or $\boxed{i_{coil} = \frac{V_{coil}}{L_{coil}s + R_{coil}} \leftrightarrow L\frac{di}{dt} + iR = V}$

(if inductance "matters")

$\boxed{V_{back} = K_T \dot{x}_{coil}}$ $\boxed{V_{coil} = V_{Amp} - V_{back}}$



Problem 5 $\quad 100\frac{dT}{dt} + 2T = 20Q \rightsquigarrow \mathcal{L}$

$$(100s + 2)\,T(s) = 20\,Q(s)$$

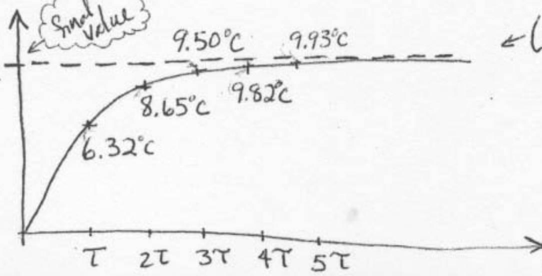
first-order system

$$\frac{\text{Output}}{\text{Input}} = \frac{T(s)}{Q(s)} = \frac{20}{100s+2} = \frac{10}{50s+1} = \frac{K}{\tau s+1}$$

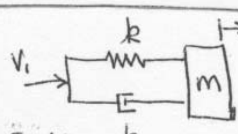
(a) $\tau = 50\,s$

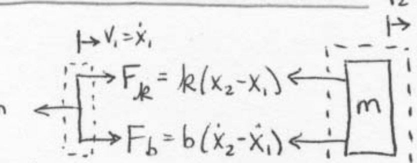
(b) $K = 10\ (^\circ C/\text{Watt})$

(c) TF: $\frac{T(s)}{Q(s)} = \frac{10}{50s+1}$



Problem 6





In general, to get EoM:
Balance forces at all masses & massless nodes...

at the mass: (Newton's 2nd Law...) $\sum F_m = m\ddot{x}_2 = -k(x_2 - x_1) - b(\dot{x}_2 - \dot{x}_1)$

Yes, there is a required external force "F_{in}" at the left on the "massless node" where V_1 is — but we have enough info to relate V_1 and V_2 already: ($\div m \rightarrow$)

$$\ddot{x}_2 = -\frac{k}{m}x_2 + \frac{k}{m}x_1 - \frac{b}{m}\dot{x}_2 + \frac{b}{m}\dot{x}_1$$

if we want wrt V, not x! $\frac{d}{dt}$

$$\ddot{V}_2 = -\frac{k}{m}V_2 + \frac{k}{m}V_1 - \frac{b}{m}\dot{V}_2 + \frac{b}{m}\dot{V}_1$$

Given:

output: $y = V_2 = \dot{x}_2$ ... A logical set of state variables would
input: $u = V_1 = \dot{x}_1$ include x_1, x_2 and $\dot{x}_2$. (Not $\dot{x}_1$, b/c no mass)

You can put your state var's in any order to create the state vector, $\underline{x}$, BUT you must then be consistent in creating the state eqn's correctly!

I choose:

$$\underline{x} = \begin{bmatrix} x_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix},\ \therefore\ \dot{\underline{x}} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \ddot{x}_2 \end{bmatrix} \rightarrow$$

(a)

$$\boxed{\begin{aligned} \frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix} &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ \frac{k}{m} & -\frac{k}{m} & -\frac{b}{m} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ \frac{b}{m} \end{bmatrix}u \\ y &= \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix} + [0]u \end{aligned}}$$

$u = \dot{x}_1$

$y = \dot{x}_2$

Problem 6, cont'd

Recall: EOM $\rightarrow$ $m\ddot{x}_2 = -k(x_2 - x_1) - b(\dot{x}_2 - \dot{x}_1)$

$$m\ddot{x}_2 + b\dot{x}_2 + kx_2 = b\dot{x}_1 + kx_1 \quad \mathcal{L}$$

$$(ms^2 + bs + k)\,X_2(s) = (bs + k)\,X_1(s)$$

(b) $G(s) = \dfrac{Y(s)}{U(s)} = \dfrac{V_2(s)}{V_1(s)} = \dfrac{s \cdot X_2(s)}{s \cdot X_1(s)} = \boxed{\dfrac{(bs+k)}{(ms^2+bs+k)} = G(s)}$

using $b = 10$ N/m/s

char. eq: $ms^2 + bs + k = 0$

or: $s^2 + \frac{b}{m}s + \frac{k}{m} = 0$

std. form: (2nd order sys) $s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$

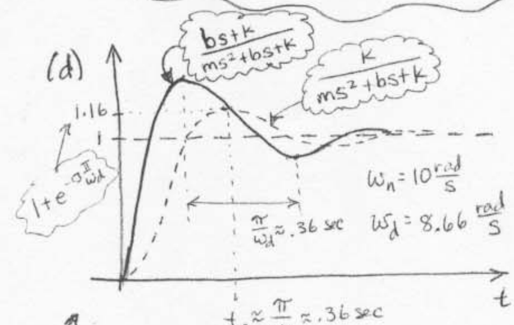
not Hz!

(c) $\omega_n = \sqrt{\frac{k}{m}} = \sqrt{100} = \mathbf{10}\ \frac{rad}{s}$ if $b=10$

$\zeta = \frac{b}{2m\omega_n} = \frac{10}{20} = \mathbf{.5}$

2% settling time $\approx \frac{4}{\zeta\omega_n} = \mathbf{0.8}$ sec

(d)



Step response is a little TRICKY b/c of the zero! w/o zero, init slope must be zero & you get familiar DASHED plot, but W/ zero, init slope $= \lim_{s\to\infty}\left(s\frac{(bs+k)}{(ms^2+bs+k)}\right) = \frac{b}{m} = 10$

in MATLAB

```
b = tf([10, 100], [1, 10, 100])
figure(2)
step(b)
```

using $b = 100$ N/m/s

$ms^2 + bs + k = 0$

$s^2 + \frac{b}{m}s + \frac{k}{m} = 0$

$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$

not Hz

if $b=100$ (c) $\omega_n = \sqrt{\frac{k}{m}} = \mathbf{10}\ \frac{rad}{s}$

$\zeta = \frac{b}{2m\omega_n} = \frac{100}{20} = \mathbf{5}$

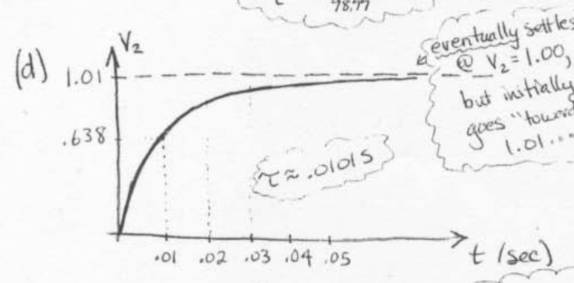
2% settling time $\approx 4\tau \approx \mathbf{.04}$ sec

(pole & zero nearly cancel, leaving influence from pole @ $s = -98.99$)

$s = -1.01$ pole, $s = -1$ zero

"τ" $\approx \frac{1}{98.99} \approx .01$ sec

(d)



$G(s) = \dfrac{100\,(s+1)}{(s+98.99)(s+1.01)} \approx \dfrac{100}{(s+98.99)}$ — $\tau \approx \frac{100}{99} \approx .0101$ s, $K \approx \frac{100}{99} = 1.01$

"nearly" cancel out...

in MATLAB

```
a = tf([100, 100], [1, 100, 100])
figure(1)
step(a)
```

Problem 6, cont'd

Note: If you are not familiar w/ MATLAB yet, you will/should be soon! You may wish to try the plotting commands at the bottom of the previous page to see better plots...

using $b = 10 \frac{N}{m/s}$

(e) zeros:

$bs + k = 0$

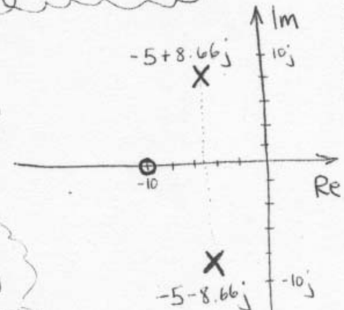
$s = \frac{-k}{b} = -10 \frac{rad}{s}$

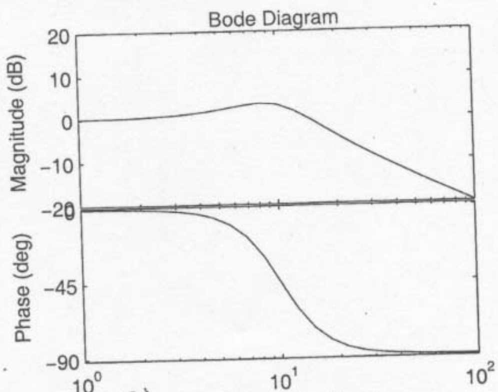
poles:

$ms^2 + bs + k = 0$

$s = -5 \pm 8.66j$

$\sigma = 5$ $\quad \omega_d = 8.66$





(f)

(g) $u(t) = v_1(t) = 10 \frac{m}{s} * \sin(5t)$

Replace "s" with "$j\omega$" in the TF, where $\omega = 5 \frac{rad}{s}$ is the input freq.

MATLAB →

```
clear j
s = 5*j
m = abs((10*s+100)/(s*s+10*s+100))
p = phase((10*s+100)/(s*s+10*s+100))
```

$m = 1.2403$

$p = -.1244 \frac{rad}{s}$

then $v_2(t) = \underbrace{12.403}_{10*m} \frac{m}{s} * \sin(5t \underbrace{-.1244}_{p\ (\text{in rad})})$

using $b = 100\ N/m/s$

(e) zeros:

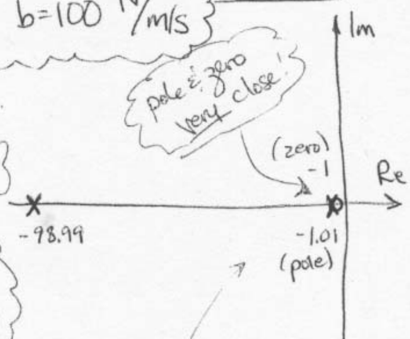
$bs + k = 0$

$s = \frac{-k}{b} = -1 \frac{rad}{s}$

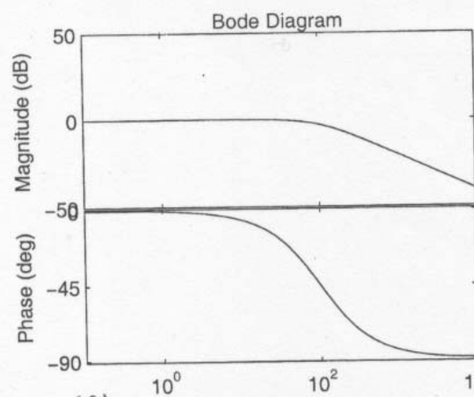
poles:

$ms^2 + bs + k = 0$

$s = -1.01, -98.99$



Having the pole & zero so close, they nearly cancel one another out! Conceptually, when $b = 100\ N/m/s$, this is such a large value, we do not "see" the effect of the (weak) spring.



(f)

(g) $u(t) = v_1(t) = 10 \frac{m}{s} * \sin(5t)$,

then $\boxed{v_2(t) \approx 10 \frac{m}{s} * \sin(5t)}$, because $5 \frac{rad}{s}$ is more than one decade below the effective "breakpoint" at $100 \frac{rad}{s}$, so not decr. in magn or phase shift, essentially.