

Department of Mechanical Engineering  
Massachusetts Institute of Technology  
2.14 Analysis and Design of Feedback Control Systems  
Fall 2004  
Assignment #4  
Distributed Wednesday September 29  
Due: **Wednesday, October 13** by 5pm in 35-231 Drop Box

Reading: Reading:  
Nise 6.1 - 6.4  
7.1-7.7

**Problem 1**

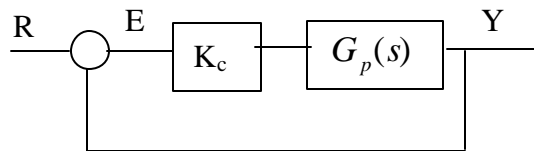
Nise Problem 5-1 part a) only

Nise Problem 5-15

Nise Problem 5-50

**Problem 2**

Determine the range of gains ( $K_c$ ) for each of the following transfer functions. In each case assume the system is a unity feedback control system as shown below:



- a)  $G_p(s) = \frac{s^2 + 4s + 100}{s^3 + 6s^2 + 11s + 6}$
- b)  $G_p(s) = \frac{20}{s^3 + 7s^2 - 28s + 20}$
- c)  $G_p(s) = \frac{4}{s^4 + 6s^3 + 13s^2 + 12s + 4}$

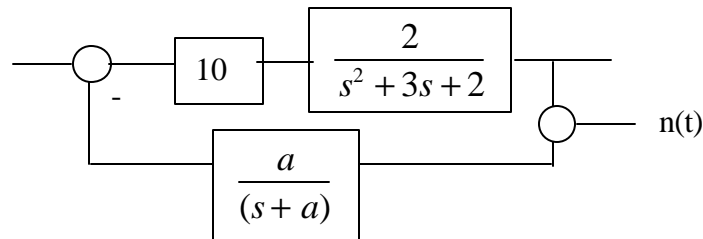
In each case please also:

Determine if the system is open-loop stable.

Determine the open-loop poles and zeros and plot them on an s-plane.

### Problem 3

We can look at the effect of specific pole or zero locations and their effect on stability using the Routh-Hurwitz method. In this case, consider the problem of designing a feedback filter for a second order system, as shown below. The plant transfer function  $G_p(s)$  is already fixed as is the controller gain, as shown. Now we need a "noise" filter in the feedback path, and want to know the tradeoff between degree of filtering (via the values of  $a$ ) and the stability.



- Accordingly, find the range of stable filter poles ( $a$ ) for this system
- Assume a noise input  $n(t) = N\sin(\omega t)$  entering as shown in the figure. Given the limits on  $a$ , what is the lowest frequency  $\omega$  for this noise that would be reduced in magnitude to  $0.1*N$  on the system output?

### Problem 4

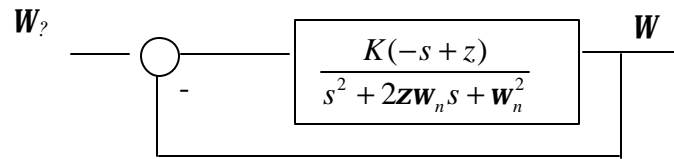
Many ships (and even small boats) exhibit somewhat odd behavior when the rudder angle is suddenly changed. Instead of turning in the expected directions, the ship will start in the opposite direction before finally turning as expected. (In other words, if you move the rudder for a right turn, it will start turning left before finally assuming the correct right turn.) This can be modeled using linear systems theory as follows:

$$G(s) = \frac{\Omega(s)}{f(s)} = \frac{K(-s + z)}{s^2 + 2z\omega_n s + \omega_n^2}$$

where  $\Omega$  is the turning rate in rad/sec and  $\phi$  is the rudder angle in radians .

- Can you give a physical explanation for this behavior? (Remember that that rudder is large and being moved in a stepwise fashion).
- Before doing any numerical analysis, explain how this transfer function would model this behavior. You may sketch the expected step response to help illustrate your answer, For that you can assume  $K = 10$   $z=2$  ,  $\omega_n = 1$  and  $z = 1$ .

c) Now consider the stability of this system if we build a closed-loop turning rate controller:



Although it is second order we want to be sure it is stable for all values of gain  $K$ .

Is it?

#### **Problem 4**

Nise Problem 7.1

Nise Problem 7.3