Department of Mechanical Engineering
Massachusetts Institute of Technology
2.14 Analysis and Design of Feedback Control Systems
Fall 2004
Assignment #9
Distributed Thursday November 17, 2004
Due: **Wednesday, November 24** by 5pm in 35-231 Drop Box

Reading: Reading: Nise Chapter 13

Problem 1

An integral controller can be realized by the transfer function

$$G_c(z) = \frac{U(z)}{E(z)} = \frac{K}{(z-1)}$$

which is based on the mapping of s=0 to z=+1.

a) Looking at the resulting difference equation to realize this controller, show how it actually represents a running sum of the input:

$$u_k = \sum_{i=1}^{k} e_i$$

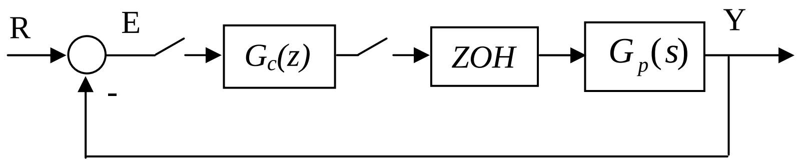
which is the discrete equivalent of

$$u(t) = \int_0^t e(t)dt$$

b) In some texts the integral controller is represented as $G_c(z) = K\frac{z}{(z-1)}$ based on the z-transform of $\frac{1}{s}$. Using difference equations and a root locus argument, discuss whether this alternative form will make any difference in either the steady – state error of the closed-loop system or the transient response.

Problem 2

Consider the position servo problem given by:



where $G_p(s) = \dfrac{1}{s(\tau s + 1)}$ and $\tau = 0.1$

Our goal is to design a controller $G_c(z)$ for this system to meet the following specs.:

- Zero error to a step input
- A settling time $\leq$ 1.0 sec
- A damping ratio $\geq$ 0.5

For this system:

a) Find the equivalent $G_p(z)$ for the plant for a sampling time of 0.05 sec.

b) Plot the root locus and see if the specifications can be met. Be sure to determine the loop gain for this design.

c) For your design point, what will be the steady state error to a ramp?

d) Now repeat this design for a sampling time of 0.2 sec. Can you still meet the specifications?

Problem 3

For the same system as in Problem 3, determine a controller that will give zero ramp error. If necessary, use a PI controller form to help in getting the best possible transient response. (Use the case of T=0.05 sec.). Feel free to use the MATLAB rltool function to aid in this design.

Can you still meet the time response specs?

Problem 4

For a system described by the plant transfer function:

$$G(s) = \frac{(s+3)}{s(s+1)(s+2)}$$

a) Find the equivalent G(z) assuming a zero order hold function at the input and T = 0.1. you will need to use partial fraction expansion to get this from the available transform tables.

b) Confirm your answer to a) using the MATLAB command c2d.

c) Plot the root locus of this system on the z-plane using MATLAB and find the "best" operating point for a gain compensated system (i.e., $G_c(z) = K_c$)

d) Now, add a pole zero pair to the controller to improve the response. What is the fastest response you can achieve such that the dominant response has a damping ratio ≤ 0.7?

e) By looking at the difference equation for the controller, show why you could not simply add a pure zero to the controller, but only a pole zero pair.