Department of Mechanical Engineering

2.14 ANALYSIS AND DESIGN OF FEEDBACK CONTROL SYSTEMS

Fall Term 2003

Problem Set 1

Assigned: Sept. 3, 2003 Due: Sept. 10, 2003

Reading:

- Nise, Chapter 1.
- Nise, Chapter 2, Sections 2.1 - 2.3

Problem 1: Nise, Ch. 1, Problem 2.

Problem 2: Nise, Ch. 1, Problem 5.

Problem 3: Use the Laplace transform to find (i) the step response, and (ii) the impulse response of a plant with transfer function

$$G(s) = \frac{1}{(s+2)(s+6)}$$

Problem 4: A "cruise-control" for an automobile is a velocity control system. Assume that a simplified model for the car consists of a lumped mass m, driven by a force source $F(t)$ (the engine), and that all resistance effects are viscous and may be expressed by a coefficient B.

(a) Derive a differential equation relating the car velocity $v(t)$ to the applied force $F(t)$.

(b) The car's mass was measured at 1000 kg. In a coast-down test from an initial velocity of 10 m/s it was found that it took 8 s to slow to a speed of 0.2 m/s. Estimate the value of the viscous coefficient B.

The cruise-control monitors the angular velocity of the drive shaft (and hence the forward velocity of the car) and forms an error signal

$$e(t) = v_{desired}(t) - v(t)$$

which is then used to control the throttle to generate the propulsive force

$$F(t) = Ke(t)$$

where K is a constant.

(c) Draw a block diagram of the closed-loop system.

(d) Derive the differential equation relating the velocity $v(t)$ to the set-point $v_{desired}(t)$.

(e) Derive the transfer function relating the velocity $v(t)$ to the set-point $v_{desired}(t)$.

(f) Find the closed-loop system time-constant if $K = 100$ n.s/m.

(g) What will be the steady-state speed of the car on level ground if $v_{desired}$ is set to 60 mph? (Assume the value for B that you found in part (b) above, and the value of K used in (f).)

(h) What will be the steady-state speed of the same car when traveling up an incline of 15° if $v_{desired}$ is set to 60 mph?

<u>**Problem 5:**</u> In the figure shown below, R is an input that the output C should follow as closely as possible, and D is an unwanted disturbance to which (ideally) C should not respond at all. Find the transfer functions relating:

(i) C(s) to R(s) (assume $D(s) = 0$), and

ii) C(s) to D(s) (assume $R(s) = 0$).

Use the results of (i) and (ii) to find the closed-loop steady-state response to unit step inputs in (iii) the input U, and (iv) the disturbance D. What happens to the two steady-state responses as the controller gain K becomes very large? Find the value of the steady-state response when $K = 80$. Does closed-loop operation reduce the effect of the disturbance?