Department of Mechanical Engineering

2.14 Analysis and Design of feedback Control Systems

Fall Term 2003

Problem Set 10

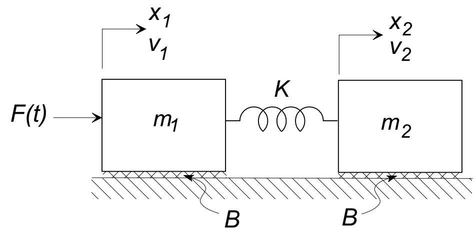
Assigned: Nov. 26, 2003 **Due:** Dec. 8, 2003

Reading: Class Handouts on State-Space Methods:
- Intro. to Matrices
- State-Space Representation of LTI Systems
- Time-Domain Solution of LTI State Equations

Nise, Chapter 3 State Space Modeling Methods
Nise, Chapter 4 (4.10, 4.11) Solution of State Equations
Nise, Chapter 5 (5.7, 5.8) Representations and Transformations in State Space
Nise, Chapter 7 (7.8) Steady-State Error for Systems in State Space
Nise, Chapter 12 Design via State Space

The purpose of these exercises is to reinforce your understanding of the state-space representation and state-feedback through the use of *Simulink*

Create a Model: Consider the simple mechanical system shown below. The task is to control the position of the mass m_2 using the external force $F(t)$ applied to mass m_1.



The two masses slide on a lubricated surface with a low, but significant viscous damping coefficient B. The two masses are coupled by a spring of stiffness K. Develop a state-space model for the plant using the following state variables: x_1, v_1, x_2, v_2, and show the model to be:

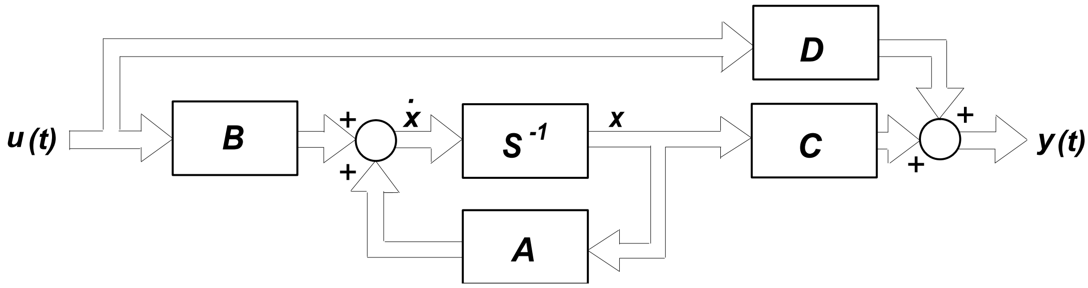
$$\begin{bmatrix} \dot{x_1} \\ \dot{v_1} \\ \dot{x_2} \\ \dot{v_2} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -K/m_1 & -B/m_1 & K/m_2 & 0 \\ 0 & 0 & 0 & 1 \\ K/m_2 & 0 & -K/m_2 & -B/m_2 \end{bmatrix} \begin{bmatrix} x_1 \\ v_1 \\ x_2 \\ v_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/m_1 \\ 0 \\ 0 \end{bmatrix} F(t)$$

$$y = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ v_1 \\ x_2 \\ v_2 \end{bmatrix}$$

You must show your working to receive credit for this part.

Assume $m_1 = 2$ kg, $m_2 = 0.5$ kg, $K = 10$ N/m, and $B = 0.01$ N.s/m.

Simulate your model: Use Simulink to make a simulation of your model as follows. You should simulate the basic state-space block diagram as shown below:



Your simulation should use only:

1. The *Integrator* block (Math Operations)
2. The *Matrix Gain* block (Math Operations)
3. The *Summer* block (Math Operations)
4. A *Signal Generator* (Sources) to drive the model
5. One or more *Scopes* (Sinks) to view you outputs

Remember that the summer and integrator blocks can use vector inputs and outputs. (Your simulation diagram should look almost the same as the figure above - without a **D** matrix.) To enter parameters into a block, double click on it and fill in the details in Matlab format. To "flip" a block (left-right) right-click on it and choose Format/Flip Block. Double-click the Signal Generator and set it for a square wave, of unit amplitude, and frequency of 0.25 Hz. Before you run your simulation enter a sensible stop time in the Simulation Parameters menu (Simulation/Simulation Parameters...). Run your simulation (Simulation/Start), record the position of mass m_2 on a scope. Print your block diagram, and a response plot. Make sure you save your model (in a ".mdl" file).

Design a Controller: Use Matlab to design a state-feedback controller that will have a pair of dominant poles with a damping ratio of 0.707, and a rise-time of 0.5 sec. (Choose where you want to place the third pole.) Use Matlab's `acker()` function to find the feedback gains.

Add Your Controller to Your Simulink Model Reload (if necessary) your model and add the state-feedback controller. Plot the square wave response.