Department of Mechanical Engineering

2.14 Analysis and Design of feedback Control Systems

Fall Term 2003

Problem Set 3 Solutions

Problem 1:

(a)

$$G(s) = \frac{a+\epsilon}{s+a} + \frac{s}{s+a}$$

(i) The step response of $1/(s+a)$ is

$$y_1(t) = \frac{1}{a}\left(1 - e^{-at}\right)$$

(ii) Differentiating $y_{step}(t)$ gives the step response of $s/(s+a)$ as

$$y_2(t) = e^{-at}$$

(iii)] The combined response is

$$y(t) = (a+\epsilon)y_1(t) + y_2(t) = \frac{a+\epsilon}{a} - \frac{\epsilon}{a}e^{-at}$$

(b) The response has the following properties:

1. $y(0^+) = 1$

2. $y(\infty) = \frac{a+\epsilon}{a}$

Clearly as $\epsilon \to 0$, the step response $y(t) \to 1$.

Problem 2: **PreLab:**

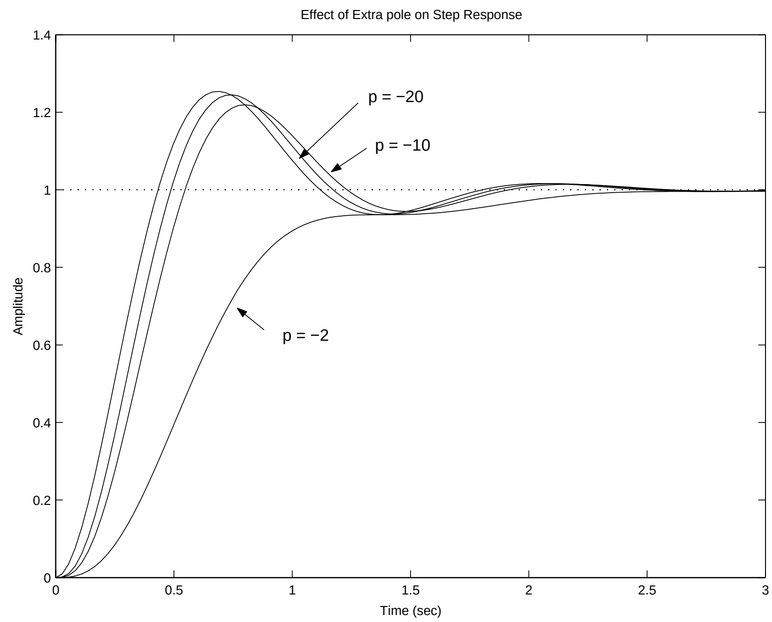
1. (a) $G(s) = \frac{25}{s^2+4s+25}$ giving $\omega_n = 5$, and $\zeta = 0.4$. From Sec. 4.6

$$\begin{aligned}
\%\text{OS} &= e^{-\zeta\pi/\sqrt{1-\zeta^2}} \times 100 = 25.3\% \\
T_s &= \frac{4}{\zeta\omega_n} = 2 \text{ sec.} \\
T_p &= \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 0.68 \text{ sec.} \\
T_r &\approx 0.29 \quad \text{sec. from Fig. 4.16}
\end{aligned}$$

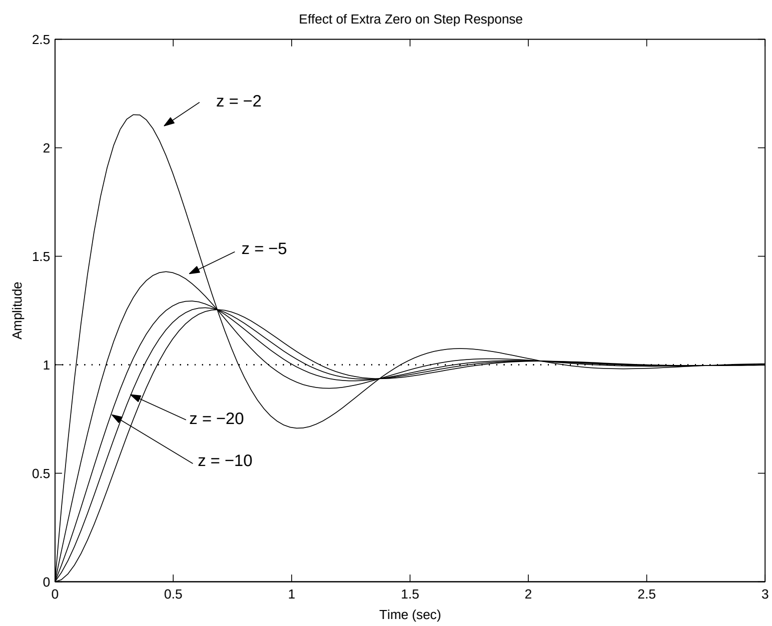
The poles are at $-2 \pm 4.5826j$.

(b) The new pole is placed deep in the right-half plane (at $s = -200$), so that any contribution to the transient response (of the form e^{-200t}) will decay very quickly and will not significantly affect the transient response.

(c) Using the argument in (b) we would expect that as the pole is moved toward the imaginary axis the effect on the step response will become more significant. The plot below shows the effect. Note that in each case the system gain was adjusted to be unity.



2. Similarly we would expect the zero closest to the imaginary axis to have the most effect on the transient response. Here is a plot to show it:

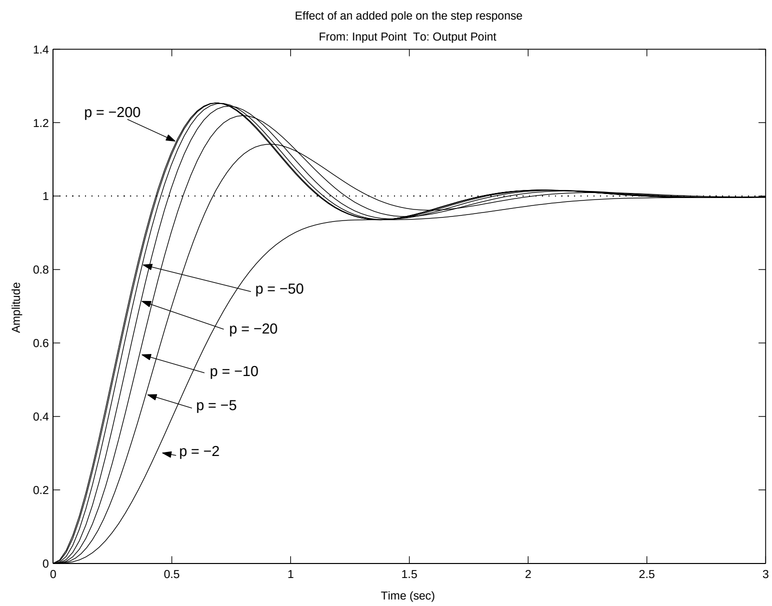


3. As $b \to a$ pole-zero cancellation will occur, as in Problem 1, and the effect of the added pole and zero will be minimized. Therefore $b = 3.01$ will have the minimum effect.

4. The answer is similar to 3 above, namely when $b \to a$ the effect of the added pole and zero will be minimum. The answer is $b = 30.01$.

Lab:

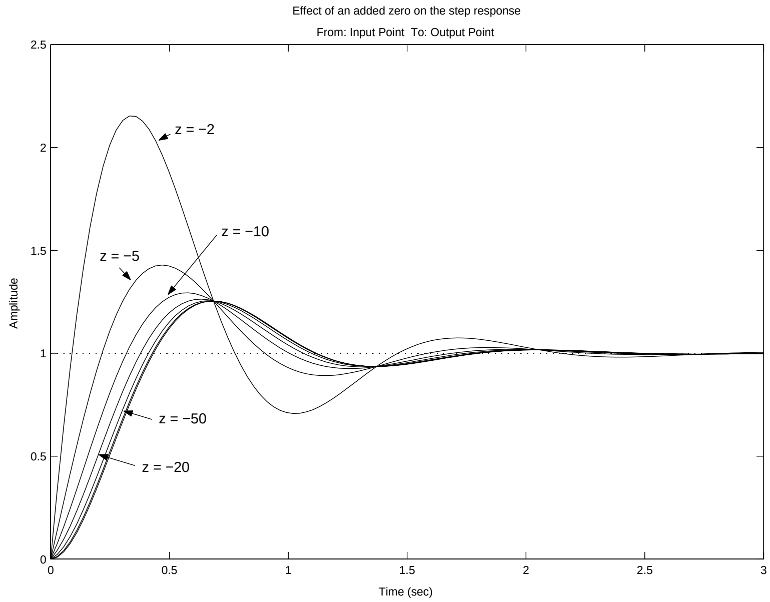
1. Simulink's LTI Viewer produced the following plot for the added pole:



The values taken from Simulink's LTIViewer were

Pole	%OS	T_p	T_r	T_s
-200	25.3	0.690	0.294	1.68
-20	25.3	0.745	0.306	1.73
-10	22	0.801	0.336	1.78
-5	14	0.992	0.408	1.83
-2	-	-	0.75	2.09

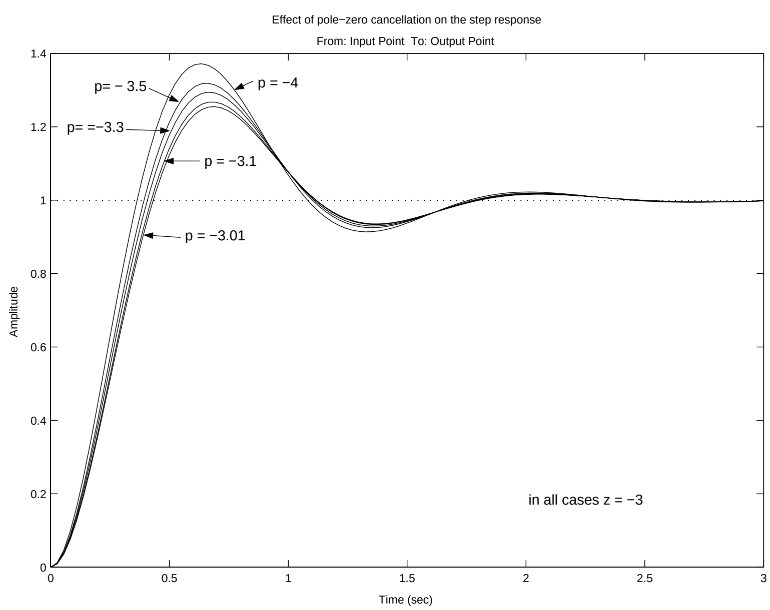
2. Simulink's LTI Viewer produced the following plot for the added zero:



The values taken from Simulink's LTIViewer were

Zero	%OS	T_p	T_r	T_s
-200	25.3	0.690	0.293	1.68
-50	26	0.663	0.291	1.66
-20	26.3	0.635	0.279	1.63
-10	29.3	0.58	0.17	2.01
-5	42.9	0.47	0.408	1.83
-2	115	0.331	0.07	2.02

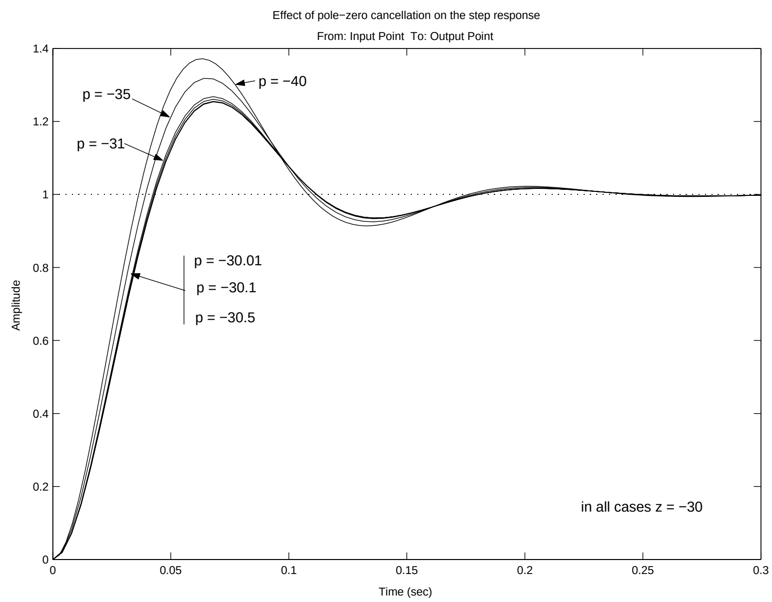
3. Simulink's LTI Viewer produced the following plot for the pole-zero proximity effect:



The values taken from Simulink's LTIViewer were

Pole	%OS	T_p	T_r	T_s
-3.01	25.5	0.690	0.293	1.68
-3.1	26.8	0.690	0.288	1.68
-3.3	29.4	0.663	0.277	1.68
-3.5	31.9	0.663	0.267	1.67
-4	37.2	0.635	0.247	2.11

4. Simulink's LTI Viewer produced the following plot for the pole-zero proximity effect:



The values taken from Simulink's LTIViewer were

Pole	%OS	T_p	T_r	T_s
-30.01	25.5	0.0690	0.0293	0.168
-30.1	25.5	0.0690	0.0293	0.168
-30.5	26.1	0.0690	0.0290	0.168
-31	26.8	0.0690	0.0288	0.168
-35	31.9	0.0663	0.0267	0.168
-40	37.2	0.0635	0.0247	0.168

PostLab:

1. An extra real pole in the system at $s = -p$ adds a component e^{-pt} to the transient response. If the pole is well to the left of the system poles on the pole-zero plot, this added component will decay much faster that the components due to the other poles, and the system response will not be greatly affected. However, if the pole is in the neighborhood of the system poles the duration of its component will be of the order of the others and the response will be affected. If the added pole lies to the right of the other system poles it will become "dominant" and its effects will persist after the other transients have decayed. This can be seen in the case for $p = -2$ in the plot in Section 1 of the lab.

2. The effect of a zero is to add a fraction of the derivative of the response (without the zero). As the zero approaches the imaginary axis the relative weight of the derivative is increased, and the transient response is changed.

3. This can be explained by arguments similar to those in Problem 1 of this problem set.