Department of Mechanical Engineering

2.14 Analysis and Design of Feedback Control Systems

Fall Term 2003

Problem Set 4 Solutions

Problem 1: Nise, Ch. 7, Problem 14 (p. 407)
Collapsing the inner loop, and cascading with the $1000/s$ block gives the open-loop transfer function as

$$G_{ol}(s) = \frac{10^5(s+2)}{s(s^2+1005s+2000)}$$

Hence the system is Type 1.

Problem 2: Nise, Ch. 7, Problem 30 (p. 410)

(a) For the inner loop:

$$G_1(s) = \frac{\frac{1}{s^2(s+1)}}{1+\frac{1}{s^3(s+1)}} = \frac{s}{s^4+s^3+1}$$

$$G_{ol}(s) = \frac{1}{s^2(s+3)}G_1(s) = \frac{1}{s(s^5+4s^4+3s^3+s+3)}$$

$$G_{cl}(s) = \frac{1}{s^6+4s^5+3s^4s^2+3s+1}$$

(b) From $G_{cl}(s)$ system is Type 1.

(c) Since the system is Type 1, $e_{ss}=0$.

(d) From $G_{cl}(s)$, $K_v = \lim_{s\to 0} sG_{ol}(s) = 1/3$. Therefore, $e_{ss} = \frac{5}{K_v} = 15$.

(e) The poles of $G_{cl}(s) = -3.0190, -1.3166, 0.3426 \pm j0.7762, -0.3495$. Therefore the system is unstable and the results of (c) and (d) are meaningless.

Problem 3: Nise, Ch. 7, Problem 48 (p. 417)
The closed-loop transfer function is

$$\frac{C(s)}{R(s)} = K_1\frac{K_2}{Js^2+Ds+K_3K_2}$$

Let the input concentration be R_0, then the steady-state response is

$$c_{ss} = R_0\frac{K_1}{K_3}$$

and the steady state error is

$$e_{ss} = R_0 - R_0\frac{K_1}{K_3} = R_0\frac{K_3-K_1}{K_3}$$

and the error may be eliminated if $K_1 = K_3$.

Problem 4: Nise, Ch. 7, Problem 49 (p. 418)
For the inner loop

$$G_1(s) = \frac{K(s+0.01)}{s^2+Ks+0.01K} \qquad \text{where} K = \frac{K_c}{J}$$

Then the open-loop transfer function is:

$$G_{ol}(s) = K\frac{(s+0.01)^2}{s^2(s^2+Ks+0.01K)}$$

and the system is Type 2.

(a) $e_{step} = 0$.

(b) $e_{ramp} = 0$.

(a) $e_{parabola} = \frac{1}{K_a} = 1/0.01 = 100$.

(d)

$$G_{cl} = \frac{K(s+0.01)^2}{s^4+Ks^3+1.01Ks^2+0.02Ks+10^{-4}K}$$

Form the Routh array:

s^4	1	$1.01K$	$10^{-4}K$
s^3	K	$0.02K$	0
s^2	$1.01K-0.02$	$10^{-4}K$	0
s^1	$\frac{0.0201K^2-0.0004K}{1.01K-0.02}$	0	0
s^0	$10^{-4}K$		

From the s^3 row we require $K > 0$.
From the s^2 row we require $K > 0.0198$.
From the s^1 row we require $K > 0.0199$.
From the s^0 row we require $K > 0$.
therefore we conclude that for stability

$$K = \frac{K_c}{J} > 0.0199$$

Problem 5: Nise, Ch. 7, Problem 53 (p. 421)
Let

$$\begin{aligned} E(s) &= F_{cmd}(s) - F(s) \\ &= F_{cmd}(s) - \frac{K_1(D_e s + K_e)}{s(s+K_1K_2)} \end{aligned}$$

or

$$E(s) = \frac{s(s+K_1K_2)}{s^2+(K_1K_2+K_1D_e)s+K_1K_e}$$

The steady-state error is given by

$$e_{ramp}(\infty) = \lim_{s\to 0} = s\left(\frac{1}{s^2}\right)\frac{s(s+K_1K_2)}{s^2+(K_1K_2+K_1K_2D_e)s+K_1K_e} = \frac{K_2}{K_e} < 0.1$$

Thus to meet the specifications we require $K_2 < 0.1K_e$. Since the closed-loop is second-order with positive coefficients, the system is always stable.