Department of Mechanical Engineering

2.14 ANALYSIS AND DESIGN OF FEEDBACK CONTROL SYSTEMS

Fall Term 2003

Problem Set 5: Solutions

Problem 1: Nise, Ch. 6, Problem 2.
Notice that there are sign changes in the characteristic polynomial, indicating that there is at least one unstable pole. Construct the Routh array:

s^5	1	4	3
s^4	-1	-4	-2
s^3	ϵ	1	0
s^2	$\frac{1-4\epsilon}{\epsilon}$	-2	0
s^1	$\frac{2\epsilon^2+1-4\epsilon}{1-4\epsilon}$	0	0
s^0	-2	0	0

If ϵ is a small positive quantity, there are three sign changes in the left column, indicating 3 rhp poles, and two lhp poles.

Problem 2: Nise, Ch. 6, Problem 28.
The closed-loop transfer function is

$$G_{cl}(s) = \frac{G(s)}{1+G(s)H(s)} = \frac{K(s^4 - s^3 + 2s + 2)}{(K+1)s^2 + 2(1-K)s + (2k+1)}$$

A sufficient condition for stability of a second-order system is that all coefficients are positive or

$$\begin{aligned} K &> -1 \\ K &< 1 \\ 2K &> -1 \end{aligned}$$

and we conclude that the system will be stable for $-\frac{1}{2} < K < 1$.

Problem 3: Nise, Ch. 6, Problem 52.
The closed-loop transfer function is

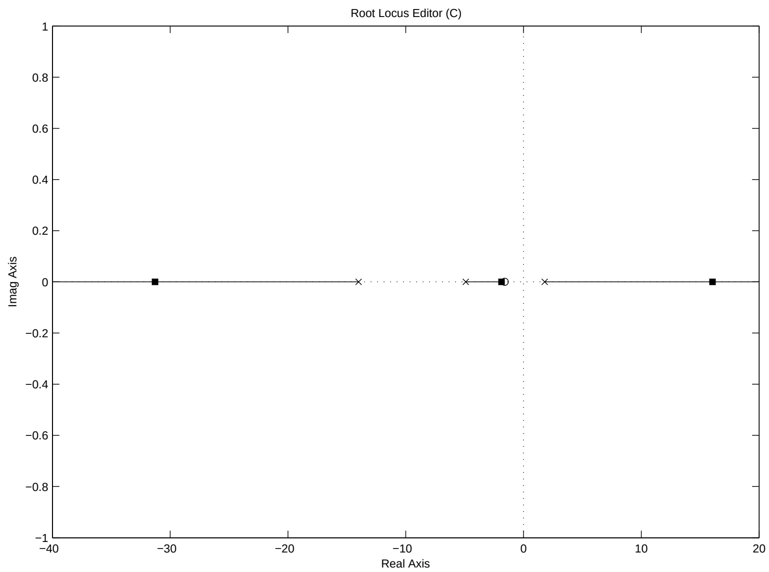
$$G_{cl}(s) = \frac{63 \times 10^6 K}{s^3 + 172.5s^2 + 4625s + (10500 + 63 \times 10^6 K)}$$

Construct the Routh array:

s^3	1	4625
s^2	172.5	$10500 + 63 \times 10^6 K$
s^1	$4564.13 - 365217.39K$	0
s^0	$10500 + 63 \times 10^6 K$	0

and we conclude that for stability $-1.67 \times 10^{-4} < K < 1.25 \times 10^{2}$.

<u>Problem 4:</u> Nise, Ch. 8, Problem 49.
Notice the negative sign in $G_2(s)$, effectively turning this into a positive feedback system. Matlab's `rlocus()` function will not handle positive feedback cases, and assumes that the gain K_2 is always positive. If you read Nise pp. 461-463 you will realize that the sketching rules change for positive feedback systems, particularly the regions on the real axis and the angles of the asymptotes. Matlab's `rltool` allows for positive feedback, so just to prove that the system is always unstable for $K_2 > 0$, I plotted the root locus below:

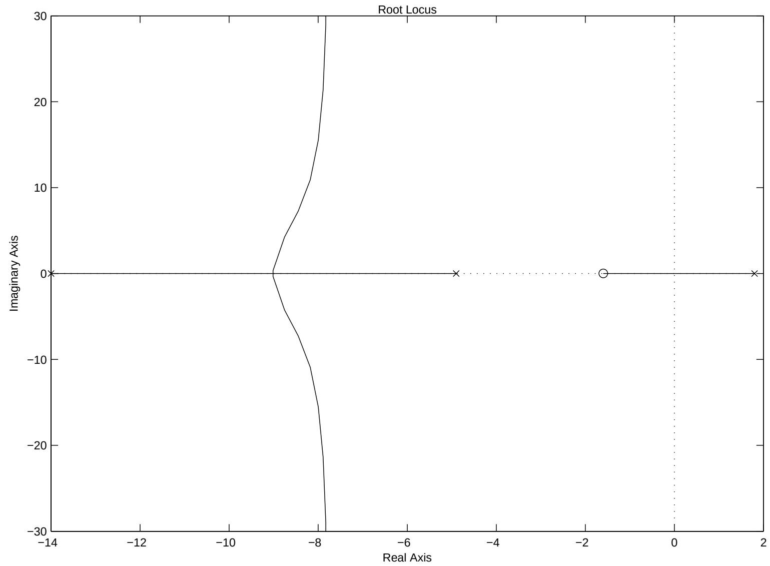


We can use the normal root locus methods if we define another gain say $K_2' = -K_2$, and know that the final value of K_2 must be negative for stability.

(a) I use rlocus() to sketch the root locus:

```
>> G2=zpk(-1.6,[-14 +1.8 -4.9],508)
Zero/pole/gain:
     508 (s+1.6)
----------------------
(s+14) (s+4.9) (s-1.8)
>> rlocus(G2)
>> sgrid
```

which generates the following plot:



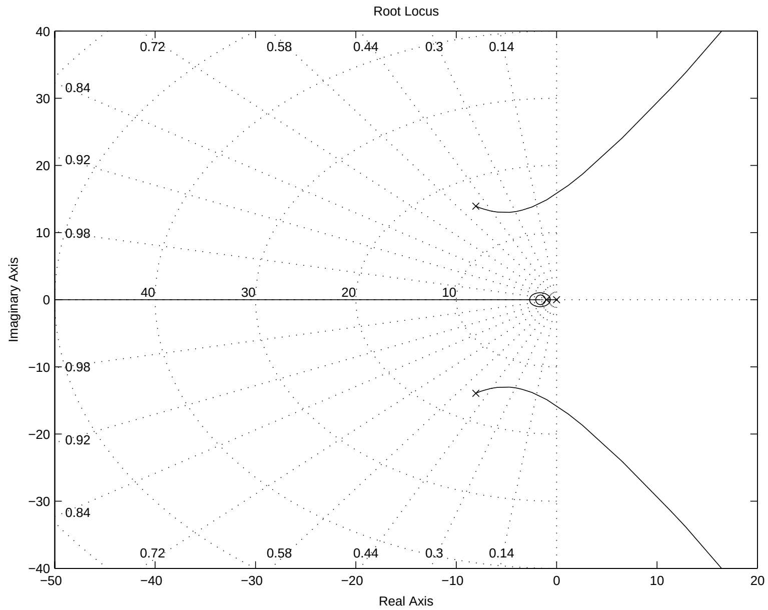
(b) I used the mouse to adjust the gain to move one the closed-loop poles on to the origin (point of instability) and read the gain as $K_2' = 0.15$. I therefore conclude that the inner loop will be stable if $K_2 < -0.15$.

(c) Using the lines on the sgrid as a guide, I use the cursor to find a point on the locus for $\zeta = 0.5$, I find that the required gain is $K_2' = 0.473$ or $K_2 = -0.473$.

(d) I use Matlab to find the closed loop transfer function of the inner loop:

```
>> inner = feedback(0.473*G2,tf(1,1))
Zero/pole/gain:
         240.284 (s+1.6)
---------------------------------
(s+1.009) (s^2  + 16.09s + 258.6)
>> outeropen = inner*tf(1,[1 0])
Zero/pole/gain:
          240.284 (s+1.6)
-----------------------------------
s (s+1.009) (s^2  + 16.09s + 258.6)
>> rlocus(outeropen)
>> sgrid
```

generating the plot:



and I used the cursor to find that the system becomes unstable at $K_1 = 16$.

(e) I used the cursor to find the point on the locus where the complex poles have a $\zeta = 0.45$. The answer is $K_1 = 2.59$.

Problem 5: Nise, Ch. 8, Problem 52.
This one is an algebraic mess! To use the root locus method we need to have a characteristic equation in the form that satisfies the angle and magnitude conditions, that is

$$1 + KG(s) = 0.$$

We can rearrange the characteristic equation of $G_{dt}(s)$ in the following form:

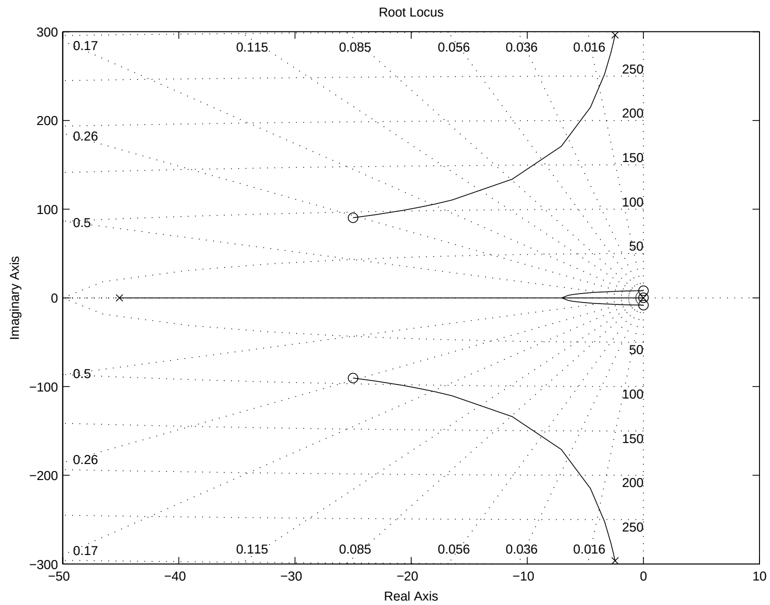
$$1 + N^2 \frac{K_{HSS}(J_R s^2 + K_{LSS})(J_G s^2 \left[\tau_{el} s + 1\right] + K_G s)}{J_R s^2 K_{LSS} \left[(J_G s^2 + K_{HSS})(\tau_{el} s + 1) + K_G s\right]} = 0$$

and use N^2 as the root locus parameter. I used Matlab to do the whole thing. The following is a ".m" file I wrote to do it. Notice that I used the `conv(a,b)` function to multiply polynomials. You can to the expansion by hand if you wish:

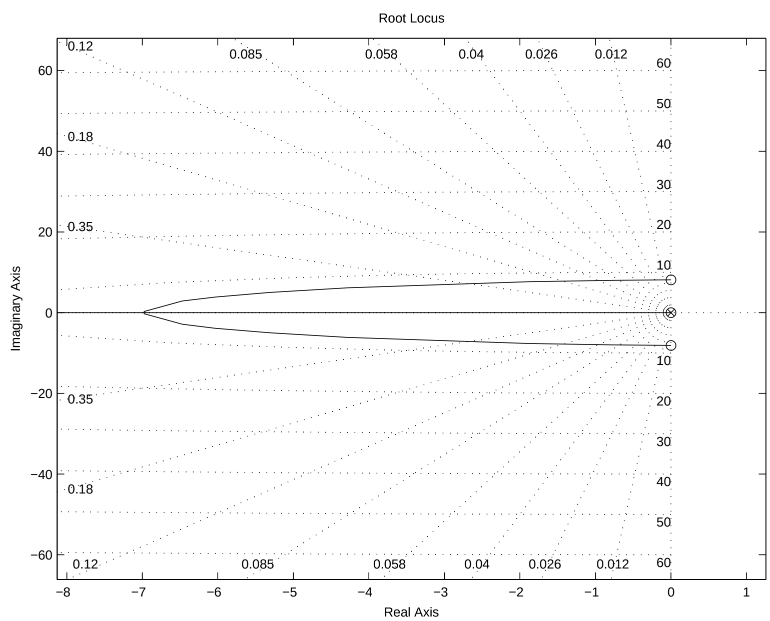
```
% 2.14 Fall 2003  Problem Set 5, Problem 5
% Nise Chapter 4, Problem 52
%
%   D. Rowell
%   10/7/03
%----------------------------------------------------------------------
echo on
% First Enter the system parameters, as defined in the problem
% (compare the symbolic and numerical transfer functions):
Khss=301e3;
```

```
Jr=190120;
Klss=12.6e6;
Jg=3.8;
tau=20e-3;
Kg=668;
%  To use the root-locus method we must write the characteristic equation
%  in the form
%                    1 + N^2 Num(s)/Den(s) = 0
% and use N^2 as the root locus parameter.  If you look at the denominator
% you'll see that we can use:
%       Num(s) = Khss(Jr*s^2 + Klss)(Jg*s^2(tau*s+1)+Kg*s)
% and   Den(s) = JrKlss*s^2*((Jg*s^2 + Khss)(Tau*s+1) +Kg*s)
% You can use any method you like to expand these out, I used the "conv"
% function to multiply polynomials.
% First define the numerator, form the coefficient arrays of the two terms
a = [Jg*tau Jg Kg 0];
b = [Jr 0  Klss];
% and convolve them to form Num(s)
Num = Khss*conv(a,b);
% The denominator is just a little trickier:
c = conv([Jg 0 Khss],[tau 1]);
c(3) = c(3)+Kg;
Den = Jr*Klss*conv([1 0 0],c);
% We're ready to go!   Form our system object from the Num and Den
% polynomials
sys=tf(Num,Den)
% and get the root locus.   Note that you will need to zoom in on the plot
% to examine the detail around the origin to find the zeta=0.5 point.
rlocus(sys)
sgrid
pause
% Using the cursor, I guessed the value of N^2 to be about 2.17x10^3
Nsquare=2.17e3;
%  Now create the overall transfer function from the equation at the top of
%  page 492 and the bits and pieces we already have:
GearTrain = tf([3.92*Klss*Khss*Kg*Nsquare 0], Nsquare*Num+Den)
figure;
% display the poles
pole(GearTrain)
% and plot the step response.
step(GearTrain)
% and note that the final value comes out to 3.92.   You can use the
% final-value theorem to show that that is correct.
%
```

(a) The following root locus plot was generated:

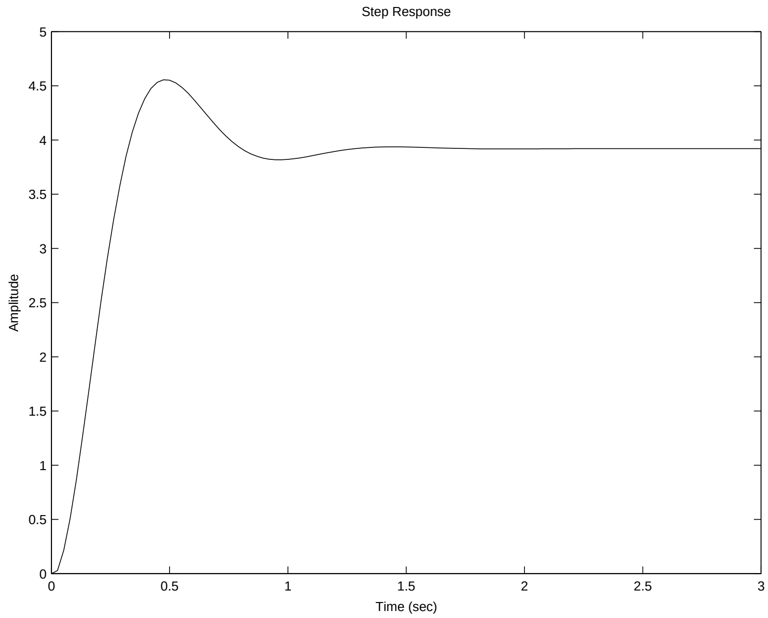


(b) I zoomed in on the origin to find the $\zeta = 0.5$point on the dominant complex poles:



On the plot I found that the required $N^2 = 2170$, or $N = 47$.

Although it was not asked for, I also plotted the step response of the system:


Step Response
Amplitude
Time (sec)
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
0
0.5
1
1.5
2
2.5
3