Department of Mechanical Engineering

2.14 ANALYSIS AND DESIGN OF FEEDBACK CONTROL SYSTEMS

Fall Term 2003

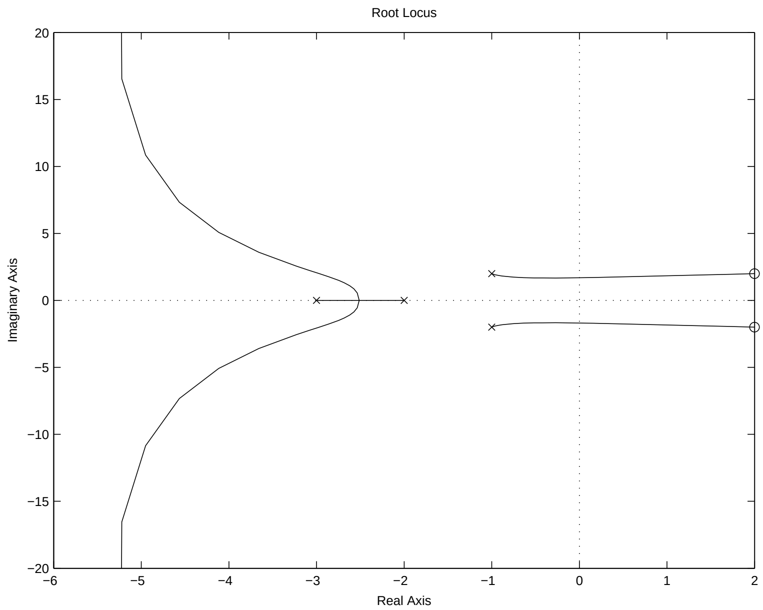
Problem Set 6: Solutions

Problem 1: Nise, Ch. 8, Problem 36.

(a) The following Matlab commands

```
sys=zpk([2+2i 2-2i],[-2 -3 -1+2i -1-2i]
rlocus(sys)
```

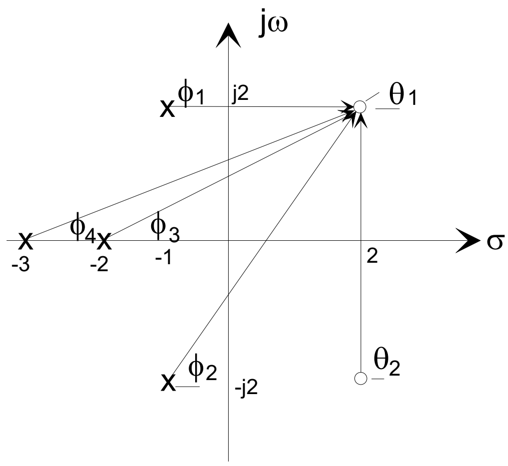
gave the plot



(b) In Matlab, searching the root locus on the imaginary axis found K = 4.26 at $\omega = \pm 1.69$ rad/s.

(c) After zooming in on the plot at the breakaway point, the breakaway is at $s = -2.51$ at a gain of $K = 0.0645$.

(d) From the pole-zero plot



and taking a test point near one of the zeros as shown, we can write

$$\theta_1 + \theta_2 - \phi_1 - \phi_2 - \phi_3 - \phi_4 = \theta_1 + 90 - 0 - \tan^{-1}\left(\frac{4}{3}\right) - \tan^{-1}\left(\frac{2}{4}\right) - \tan^{-1}\left(\frac{2}{5}\right) = 180°$$

and solving for θ_1, gives $\theta_1 = -191.5°$.

(e) There are no open-loop zeros. The closed-loop zeros are therefore the poles of $H(s)$, or $-1 \pm j2$.

(f) For a 30% overshoot in a second-order system we require $\zeta = 0.358$. This is defined by a line an angle at $\cos^{-1}(0.358) = 69.0°$ from the negative real axis. By using the command `sgrid([0.358],[3])` in Matlab, and zooming in on the intersection we find the point $-0.654 + j1.71$ and the corresponding gain $K = 0.875$

(g) The question is whether a second-order system with poles found in (f) is a close approximation to the dynamics of the closed-loop system. At $K = 0.875$ we find that there is another set of complex poles with a real part of about -2.85. This is not the five times deeper in the LHP that Nise suggests. In addition there are the closed zeros to contend with. We conclude that the a simple second-order model is not valid.

Problem 2: Nise, Ch. 8, Problem 46.
I found that the normal rlocus command in Matlab was too inaccurate for this problem. I used

```
sys = zpk([],[-500 -800 -100 0],20000)
rlocus(sys,[50000:500:120000])
```

and searched around the plot.

(a) For a settling time of 0.1 seconds, the real part of the dominant pole is $-4/T_s = -4/0.1 = -40$. Searching along root locus, we find the point 40 + j57.25 with $K \approx 102,500$.

(b) From `rlocus()` the %OS = 11%. Further, for the dominant pole the angle to the negative real axis is $\tan^{-1}(57.25/40) = 55°$, so that $\zeta = cos(55°) = 0.57$. Thus, %OS $= e^{-\zeta\pi\sqrt{1-\zeta^2}} \times 100 = 11.14\%$.

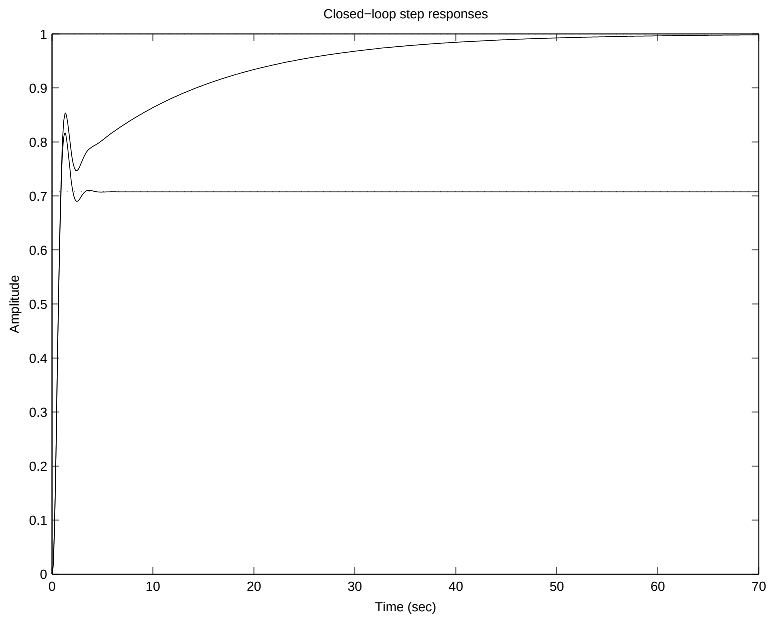
(c) The bove rootlocus range did not cross the imaginary axis. I used
`rlocus(sys,[600000:10000:800000])`.
Then looking for the $\sigma = 0$ point on the locus I found $\omega = 169.03$ and $K = 715,000$.
Therefore, for stability, $K < 715,000$.

Problem 3: Nise, Ch. 9, Problem 1.
Use

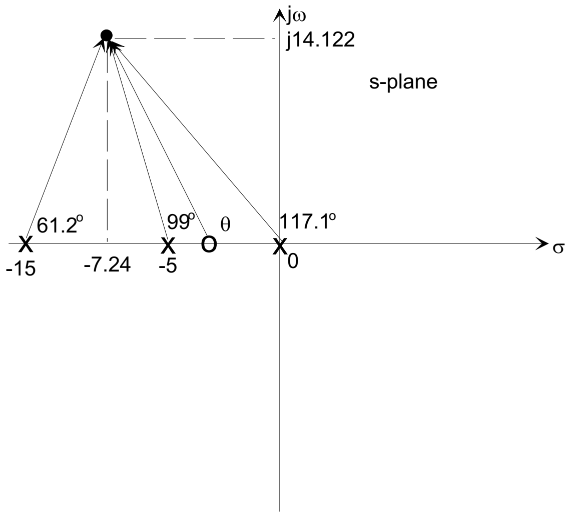
```
sys = tf([],[-1 -3 -10],1)
rlocus(sys)
sgrid([0.5],[20])
```

Uncompensated system: On the root locus, search along the $\zeta = 0.5$ line and find the operating point is at $-1.5356j2.6598$ with $K = 72.6$. Hence, %OS=$e^{-\zeta\pi\sqrt{1=\zeta*2}} \times 100 = 16.3\%$; $Ts = 4/1.5356 = 2.6$ seconds. A higher-order pole is located at -10.9285.
Compensated system: Add a pole at the origin and a zero at -0.1 to form a PI controller. Search along the $\zeta = 0.5$ line and find the operating point is at $-1.5 \pm j2.59$ with $K = 71.5$. Hence, the estimated performance specifications for the compensated system are: %OS=$e^{-\zeta\pi\sqrt{1=\zeta*2}} \times 100 = 16.3\%$; $Ts = 4/1.5 = 2.66$ seconds; static error coefficient $Kp = \infty$. Real poles are located at -0.0728 and -10.9125.

The step responses of the two closed-loop systems are shown below. Note the dominance of the pole at $s = -0.0728$ in the response with the PI controller. Is it satisfactory? Your decision...



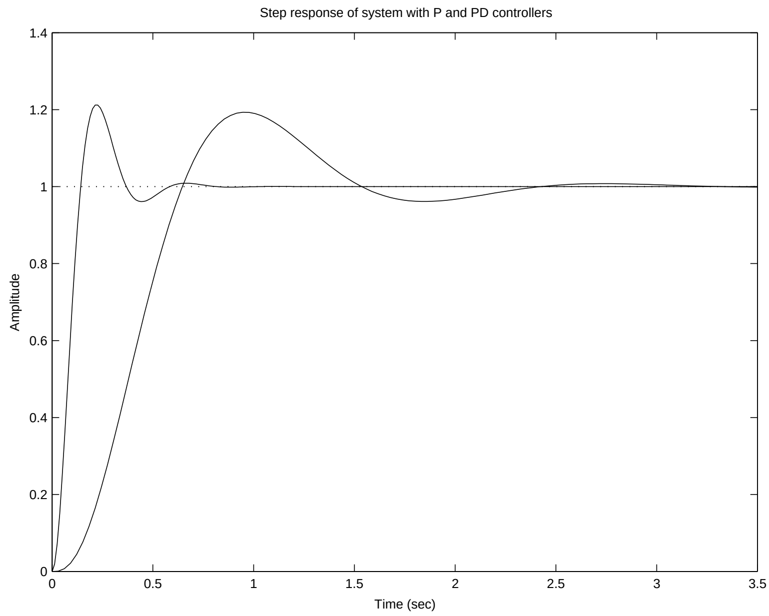
Problem 4: Nise, Ch. 9, Problem 8.
Base the answer on Example 9.7, with the results summarized in the first column of Table 9.8 (p.551) $K_1 = 257.8$, $\zeta = 0.456$, $T_s = 0.221$ s, %OS $= 10\%$, and dominant poles of $-1.809 \pm j3.531$.To improve the settling time by a factor of four we need to move the real part of the dominant roots to $4 \times -1.809 = -7.236$. To maintain the %OS we need to maintain $\zeta = 0.456$, defined by a line at an angle $\cos^{-1}(0.456)$ to the negative real axis, giving our closed-loop poles at $-7.236 \pm j14.123$. (Alternatively simply note that the ratio of imaginary part to real part of the poles will remain constant for a constant ζ, so that the imaginary part is $j(4 \times 3.531) = j14.124$.



From the pole-zero plot, the angle condition requires

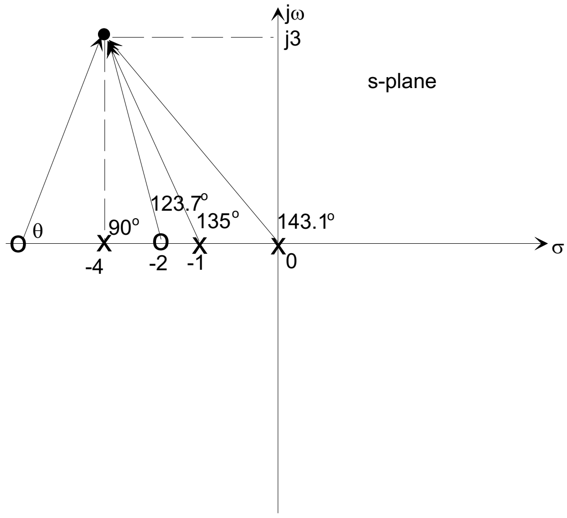
$$\theta - (61.2 + 99 + 117.1) = (2k + 1)180^\circ$$

or $\theta = 97.3^\circ$, giving the PD zero location at $s = -5.42$. Then adding the zero and using `rlocus()` we search and find that $K = 257$. The plot below compares the step response of the two closed-loop systems. Note that we have met the specifications of decreasing the settling time while maintaining the %OS.



<u>Problem 5:</u> Nise, Ch. 9, Problem 25.
If the response is described by a pair of dominant second-order poles (which we will see it isn't), the desired $T_p = 1.047$, requires the imaginary part of the closed loop poles to be $\pi/1.047 = 3$. Since $\Im/\Re = \tan(cos^{-1}\zeta)$, the magnitude of the real part will be $3/\tan(\cos^{-1} 0.8) = 4$. Hence the desired closed-loop dominant poles are at $-4 \pm j3$. First assume a PI controller $G_c(s) = (s + a)/s$ to reduce the steady-state error to zero. The choice of a is arbitrary at this point. Nise's solution chose $a = -0.1$ which gives a horrible response. Let's choose $a = -2$ and see what we get. There will be an additional zero to place (for the

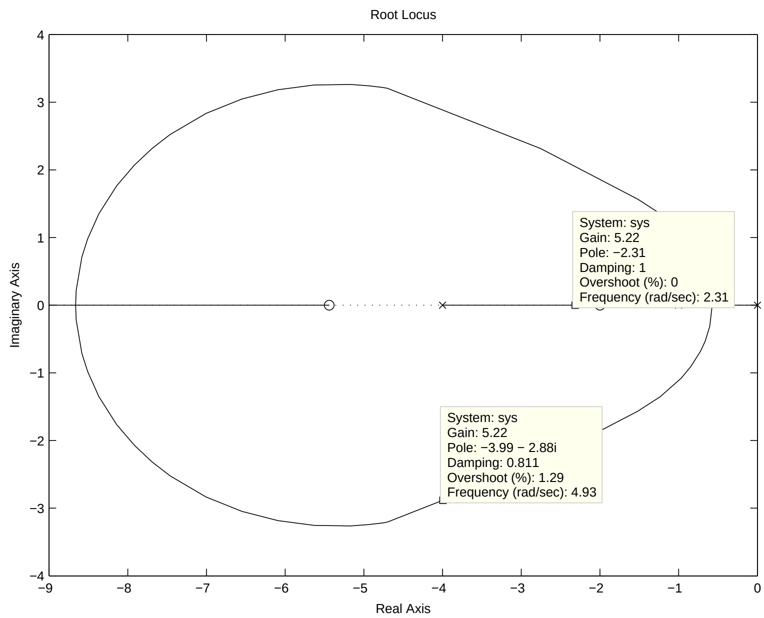
derivative control) using the angle criterion as in the previous problems. The pole-zero plot is:



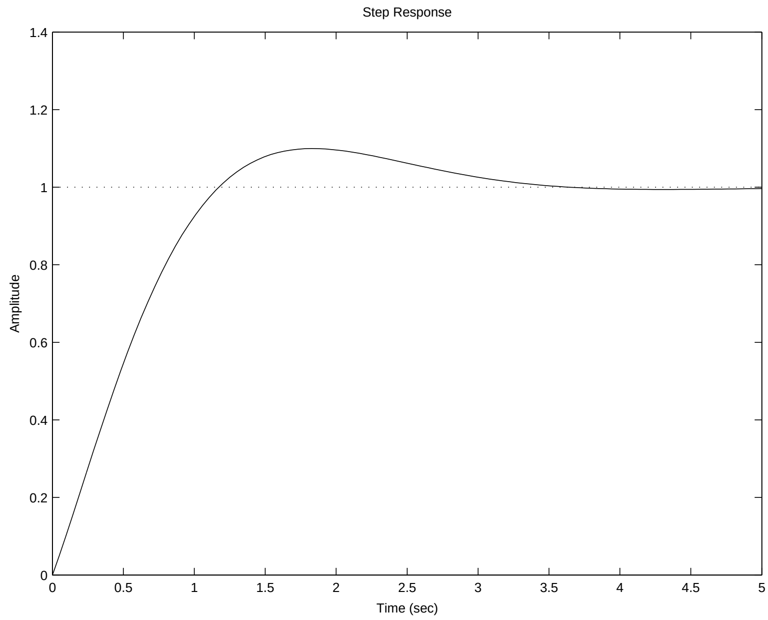
from which the angle θ contributed by the new zero must be 64.4°. This places the zero at $s = -5.44$. The PID controller is therefore

$$G_c(s) = \frac{K(s+5.44)(s+0.1)}{s}.$$

The root-locus plot is shown below:



From the root-locus plot the gain to place the poles at $-4 \pm j3$ is approximately $K = 5.22$. There is however a real pole at $s = -2.32$ at this gain, and that pole will dominate the response. The step response of the closed-loop system is shown below



which shows a peak time longer than the specification because of the dominance of the pole closer to the imaginary axis than the conjugate pair. Because we didn't meet the specification we should place the initial zero in a different position and try again... it's late and I'm tired!