Department of Mechanical Engineering

2.14 ANALYSIS AND DESIGN OF FEEDBACK CONTROL SYSTEMS

Fall Term 2003

Problem Set 7: Solutions

Problem 1: Nise, Ch. 10, Problem 1.

(a)

$$G(s) = \frac{1}{s(s+2)(s+4)} \qquad G(j\omega) = \frac{1}{-6\omega^2 + j(8\omega - \omega^3)}$$

$$|G(j\omega)| = \frac{1}{\sqrt{(6\omega^2)^2 + (8\omega - \omega^3)^2}} \qquad \angle G(j\omega) = -\tan^{-1}\left(\frac{8-\omega^2}{6\omega}\right)$$

(b)

$$G(s) = \frac{s+5}{(s+2)(s+4)} \qquad G(j\omega) = \frac{(\omega^2+40) - j\omega(\omega^2+22)}{\omega^4 + 20\omega^2 + 64}$$

$$|G(j\omega)| = \frac{\sqrt{(\omega^2+40)^2 + \omega^2(\omega^2+22)^2}}{\omega^4 + 20\omega^2 + 64} \qquad \angle G(j\omega) = -\tan^{-1}\left(\frac{\omega(\omega^2+22)}{\omega^2+40}\right)$$

(c)

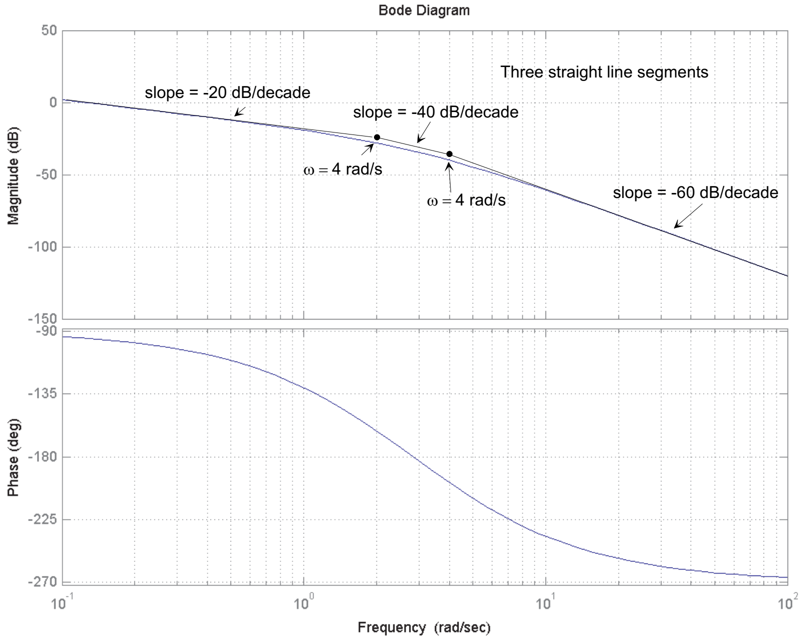
$$G(s) = \frac{(s+3)(s+5)}{s(s+2)(s+4)} \qquad G(j\omega) = \frac{-2\omega(\omega^2+13) - j(\omega^4 + 25\omega^2 + 120)}{\omega^5 + 20\omega^3 + 64\omega}$$

$$|G(j\omega)| = \frac{\sqrt{(2\omega(\omega^2+13))^2 + (\omega^4 + 25\omega^2 + 120))^2}}{\omega^4 + 20\omega^2 + 64} \qquad \angle G(j\omega) = \tan^{-1}\left(\frac{\omega^4 + 25\omega^2 + 120}{2\omega(\omega^2+13)}\right)$$

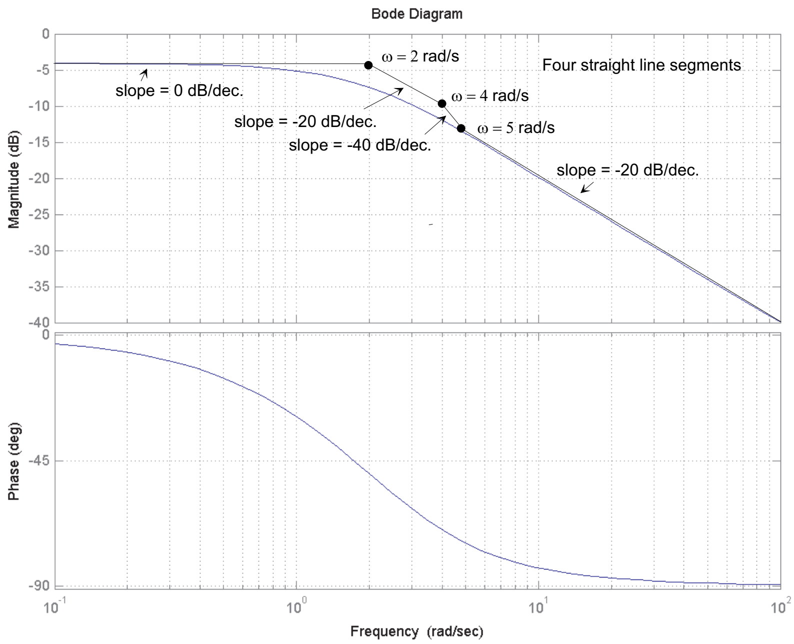
Note that because both the real and imaginary parts are negative the phase angle lies in the third quadrant.

Problems 2 and 3:

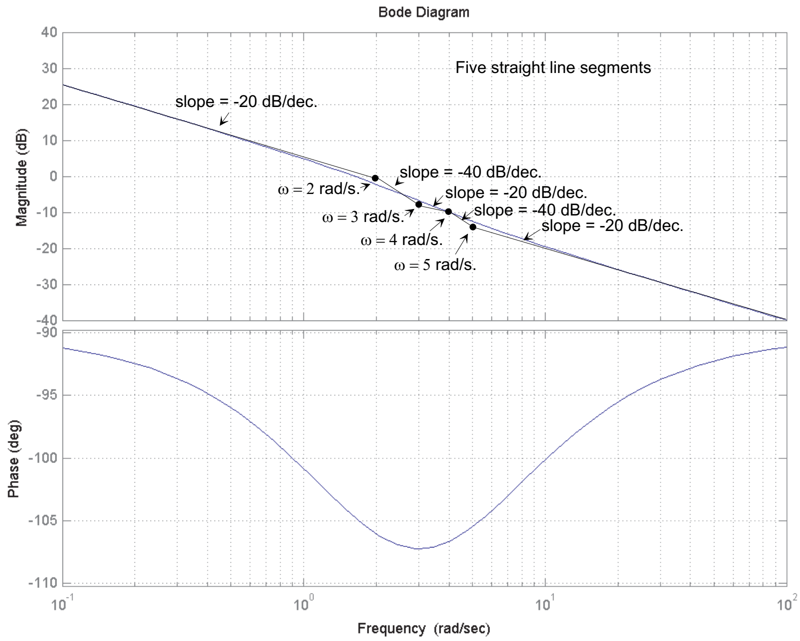
(a)



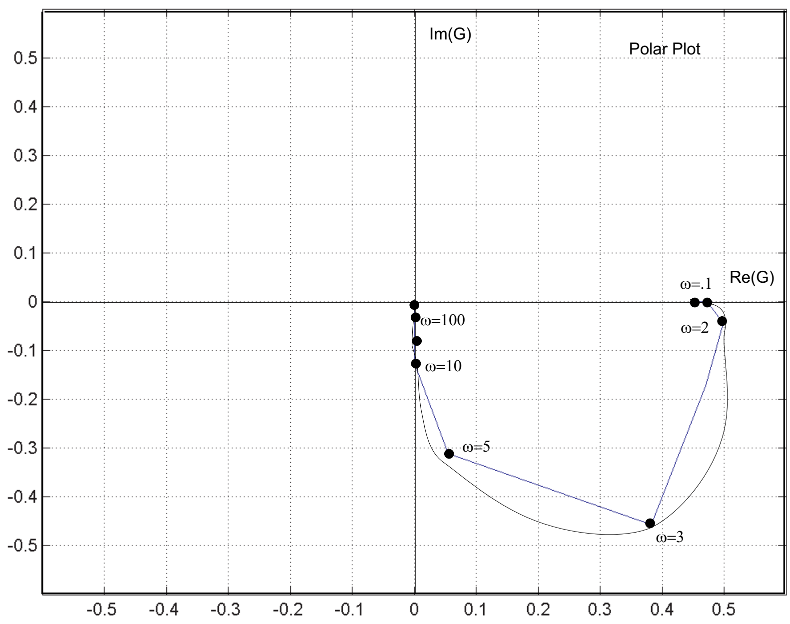
(b)



(c)



Problem 4:



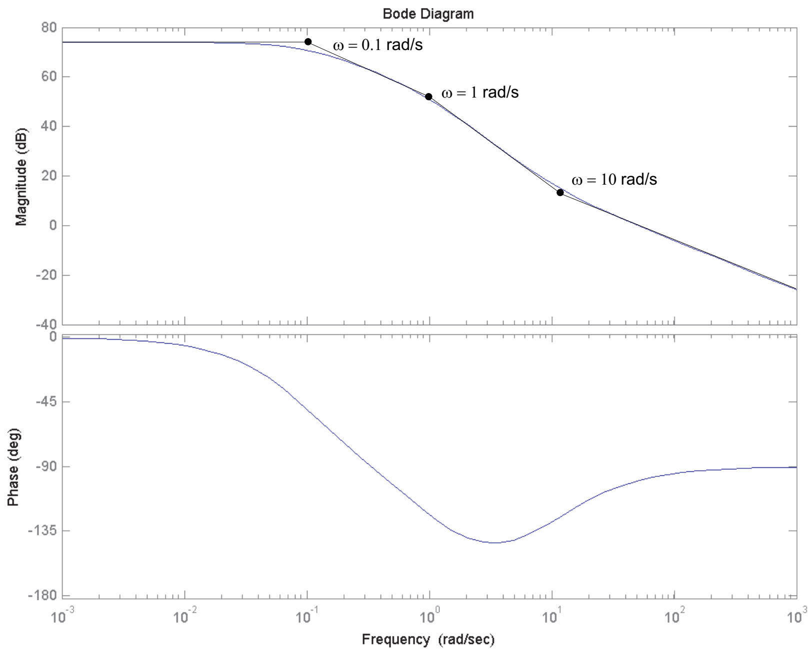
Here is how I did it: In Matlab I made two arrays, `mag` containing 13 estimates of the magnitude function, and `ang` containing estimates of the phase angles. I then used Matlab to convert and plot the values as follows:

```
linmag = 10.^(mag/20)
radians = ang.*(pi/180)
x = linmag.*cos(radians)
y = linmag.*sin(radians)
plot(x,y)
grid
```

I then imported the plot into my drawing program to tidy it up and draw a smooth curve through the points. Obviously I could have used more points in the range of 1 - 2 rad/s.

What can we say about the system? It has a slight resonant peak, indicating that it is at least a second-order system, but the high frequency phase approaches $-\pi/2$, indicating that $n - m = 1$. We might conjecture that it is a second order system with a single real zero.

Problem 5:



By placing lines with slope increments of 20 dB/decade on the magnitude curve, I determined that there were three break frequencies at 0.1 rad/s, 1 rad/s, and 10 rad/s and therefore estimated that the transfer function is of the form

$$G(s) = K\frac{s + 10}{(s + 0.1)(s + 1)}.$$

At low frequencies the gain is stated to be 54 dB or 500. Then as $s \to 0$ we have $500 = 100K$ or $K = 5$. Then

$$G(s) = \frac{5(s + 10)}{(s + 0.1)(s + 1)}$$

and the differential equation is

$$\frac{d^2y}{dt^2} + 1.1\frac{dy}{dt} + 0.1y = 10\frac{du}{dt} + 50u$$