Department of Mechanical Engineering

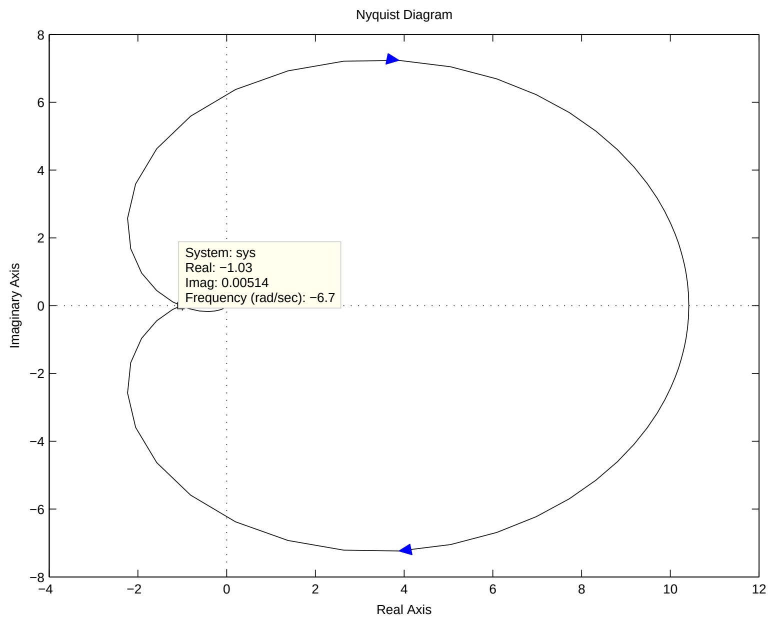
2.14 ANALYSIS AND DESIGN OF FEEDBACK CONTROL SYSTEMS

Fall Term 2003

Problem Set 8: Solutions

Problem 1: Nise, Ch. 10, Problem 10.

(a) I assumed $K = 500$ and used Matlab to make the following Nyquist plot. I then zoomed in on the plot and found that the curves crossed the real axis at approximately -1.02. Then if $K \approx 500/1.02 = 490$ the system will be stable. Since $P = 0$, we require $N = 0$, and we can assume the system will be stable for $0 < K < 490$.



Alternatively

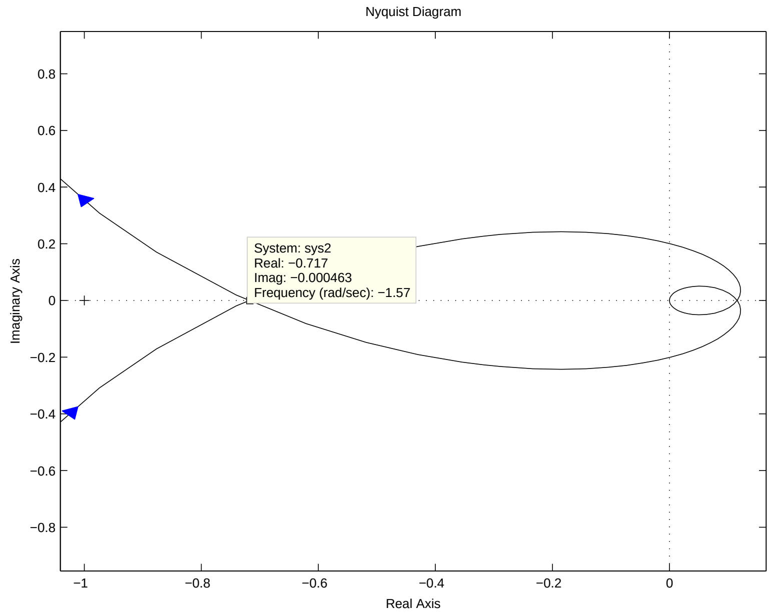
$$G(j\omega) = \frac{K}{(48 - 12\omega^2) + j(44\omega - \omega^3)}$$

which is real for $\omega = 0, \pm\sqrt{44}$. When $\omega = \sqrt{44}$, $G(j\omega) = -K/480$, so that

$$G(j\omega) = -1 \quad \text{when} \quad K = 480$$

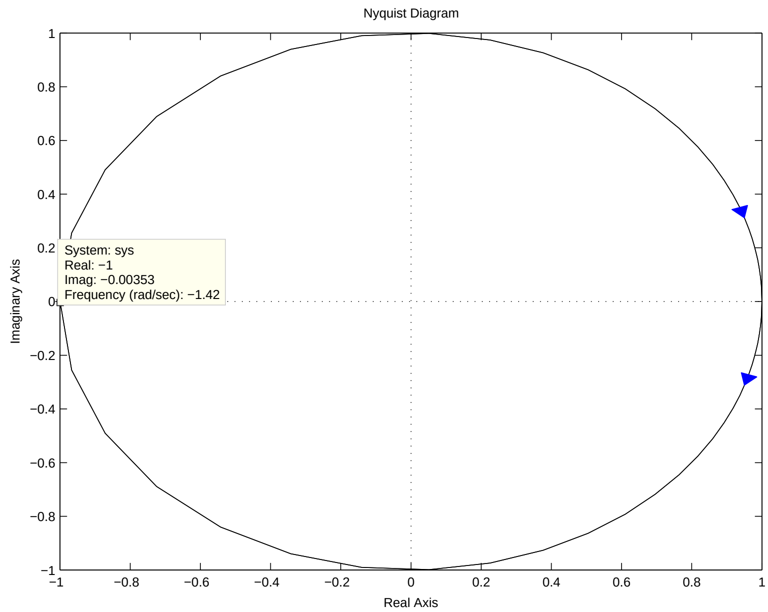
Note that the graphical estimation methods using Matlab are based on straight-line curve segments.

(b) I plotted the Nyquist plot in Matlab with $K = 1$, then zoomed in on the plot to get the following figure



The plot crosses the real axis at $K = -0.717$. Using the arguments developed in class for handling the pole at the origin, we conclude that the closed-loop system will be stable for $K < 1/0.717 = 1.39$.

(c) The Nyquist plot for $K = 1$ is shown below. Notice that it is a circle - think about what that implies for the magnitude of the frequency response.



In fact if you go through the detailed plotting method you will find that it is a double encirclement of the origin at a unit radius. Since $P = 0$, we require $N = 0$ for stability, and with $K = 1$ we are at the boundary of stability. We conclude that the closed-loop system will be stable for $0 < K \leq 1$. If $K > 1$ there will be two encirclements of the (-1,0) point and two closed-loop poles in the rhp.

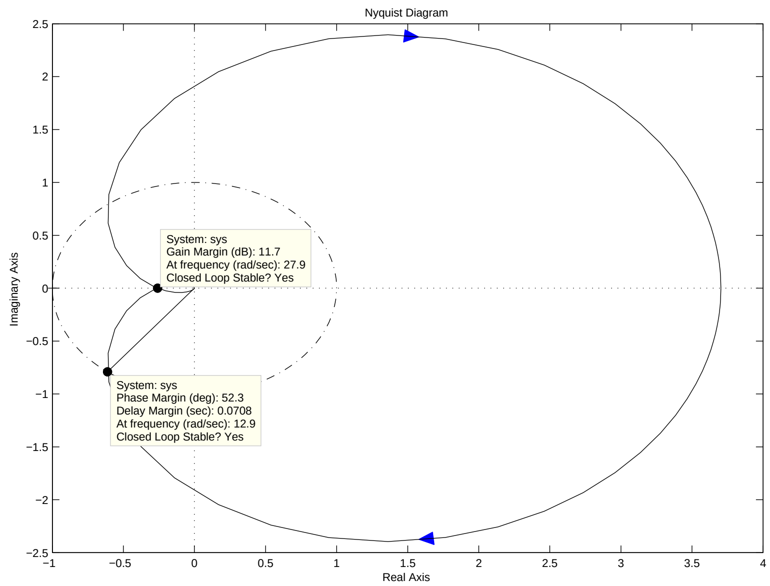
Problems 2: Nise, Ch. 10, Problem 11.
You can use the function `margin(sys)` in Matlab to do this. Note that you can have this function plot the Nyquist plot and label the gain and phase margins. Alternatively you can plot the Bode plots and estimate the margins. Here are the results:

	$K = 1000$	$K = 100$	$K = 0.1$
System 1	GM = -6.4 dB (unstable) PM = $-20.25°$	GM = 13.6 dB PM = 72.1°	GM = 73.6 dB PM = ∞
System 2	GM = -57.1 dB (unstable) PM = 90.57°	GM = -37.1 dB (unstable) PM = 95.73°	GM = 22.8 dB PM = 80.21°
System 3	GM = -60 dB (unstable) PM = ∞	GM = -40 dB (unstable) PM = ∞	GM = 20 dB PM = ∞

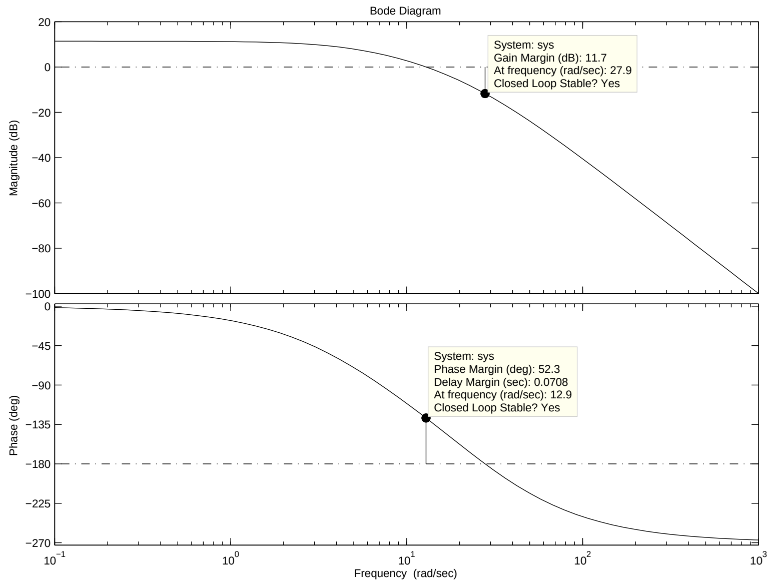
Problems 3: Nise, Ch. 10, Problem 13.
Enter the following in Matlab:

```
sys = zpk([],[-5 -18 -30],10000)
ltiview
```

Then select the system and the Nyquist plot. Right click on the plot, select Characteristics → Stability(All crossings). Then if you click on the two blue dots you will see the following:



which includes all of the information. Repeat the procedure for the Bode plot:



Problems 4:

System (a): $n = 2, m = 1$.

(a) Highest frequency break point: (Defined by the pole/zero with the greatest radial distance from the origin) 5 rad/s. High frequency slope = $(n - m) \times -20$ dB/decade = -20 dB/decade.

(b) $\lim_{\omega\to\infty} \angle G(j\omega) = (n - m) \times (-90)^\circ = -90^\circ$.

(c) Since the system has no poles or zeros at the origin, The low frequency gain is finite.

(d) From the pole-zero plot the low frequency phase shift is 0°.

System (b): $n = 3, m = 1$.

(a) Highest frequency break point: (Defined by the pole/zero with the greatest radial distance from the origin) = $\sqrt{100^2 + 120^2} = 156.2$ rad/s. High frequency slope = $(n - m) \times -20$ dB/decade = -20 dB/decade.

(b) $\lim_{\omega\to\infty} \angle G(j\omega) = (n - m) \times (-90)^\circ = -180^\circ$.

(c) Because there is a pole at the origin, the low frequency gain is infinite.

(d) The pole at the origin causes the low frequency phase shift to be -90°.

System (c): $n = 4, m = 2$.

(a) Highest frequency break point: (Defined by the pole/zero with the greatest radial distance from the origin) = $\sqrt{3^2 + 1^2} = 3.162$ rad/s. High frequency slope = $(n - m) \times -20$ dB/decade = -40 dB/decade.

(b) $\lim_{\omega\to\infty} \angle G(j\omega) = (n - m) \times (-90)^\circ = -180^\circ$.

(c) Because there is a zero at the origin, the low frequency gain is zero.

(d) The zero at the origin, and the rhp zero causes the low frequency phase shift to be 270°.

System (d): $n = 3, m = 2$.

(a) Highest frequency break point: (Defined by the pole/zero with the greatest radial distance from the origin) = 12 rad/s. High frequency slope = $(n - m) \times -20$ dB/decade = -20 dB/decade.

(b) $\lim_{\omega \to \infty} \angle G(j\omega) = (n - m) \times (-90)^\circ = -90^\circ$.

(c) Because there are no poles or zeros at the origin, the low frequency gain is finite.

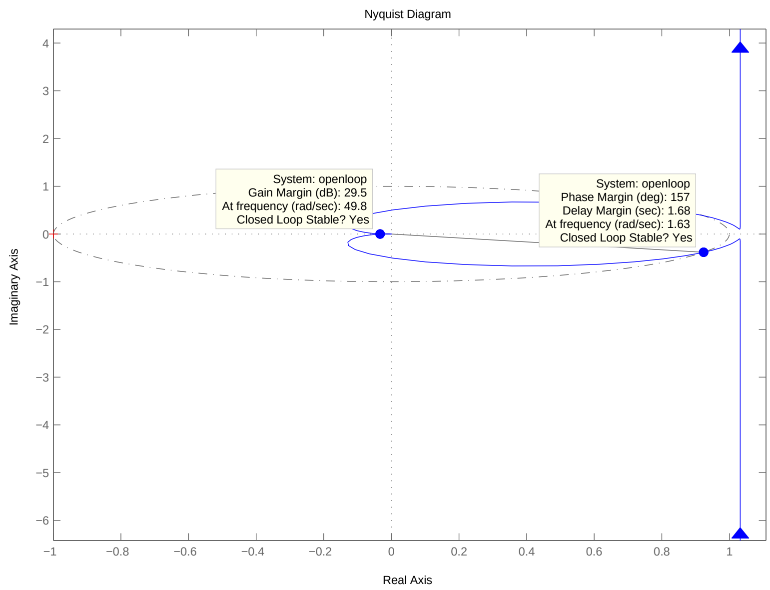
(d) The low frequency phase shift is 0°.

Problems 5: Nise, Ch. 10, Problem 34.

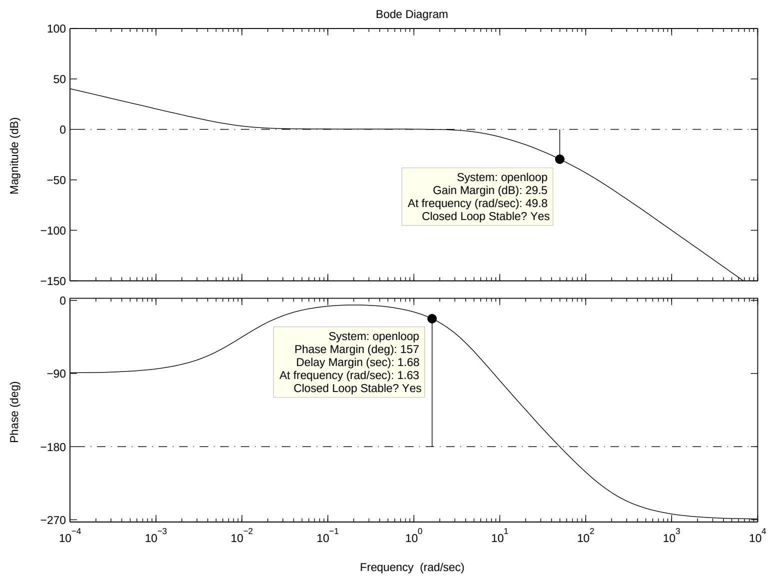
(a) Using LTIView in Matlab:

```
 control = zpk([-0.01 -6].[0 -20 -100],1000)*tf([10],[1 10 29])
ltiview(control)
```

and plotting with the Nyquist plot and stability crossings shown, the results are shown below.



The same results can be obtained from the Bode plots in the LTIviewer:

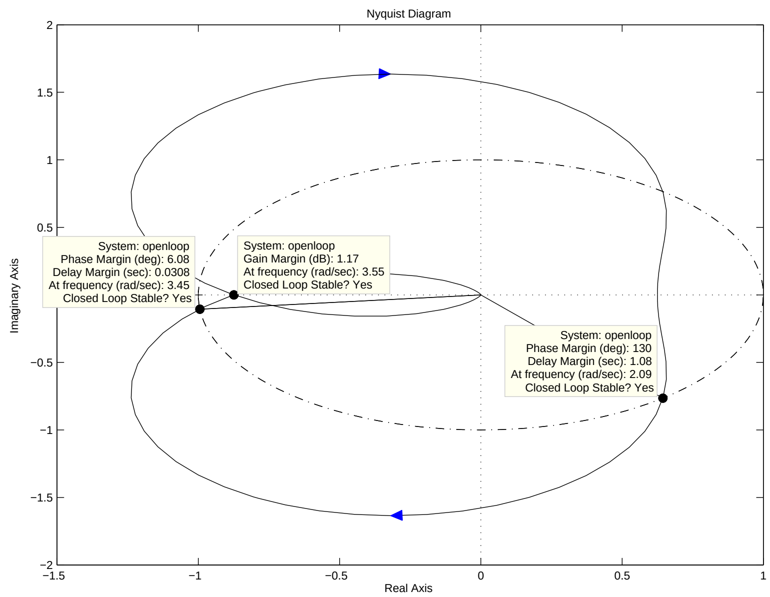


(b) The system is stable because of positive gain and phase margins.

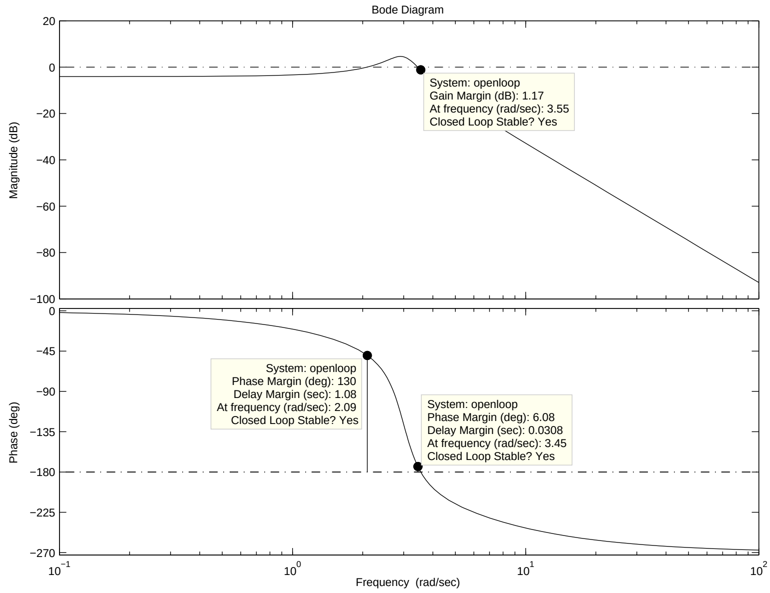
<u>Problems 6:</u> Nise, Ch. 10, Problem 38.
Using LTIView in Matlab:

```
openloop=5*zpk([],[-4],0.5)*tf([9],[1 0.9 9])
ltiview(openloop)
```

and plotting with the Nyquist plot and stability crossings shown, the results are shown below.



The gain and phase margins are shown on the plots. The same information is contained in the Bode plots below:


Bode Diagram
Magnitude (dB)
20
0
−20
−40
−60
−80
−100
System: openloop
Gain Margin (dB): 1.17
At frequency (rad/sec): 3.55
Closed Loop Stable? Yes
Phase (deg)
0
−45
−90
−135
−180
−225
−270
System: openloop
Phase Margin (deg): 130
Delay Margin (sec): 1.08
At frequency (rad/sec): 2.09
Closed Loop Stable? Yes
System: openloop
Phase Margin (deg): 6.08
Delay Margin (sec): 0.0308
At frequency (rad/sec): 3.45
Closed Loop Stable? Yes
10^-1
10^0
10^1
10^2
Frequency (rad/sec)