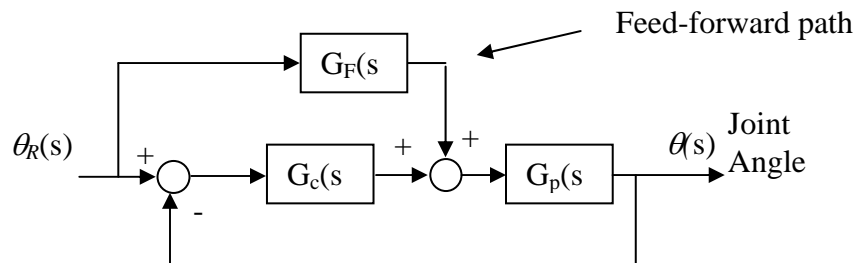


Department of Mechanical Engineering
 Massachusetts Institute of Technology
 2.14 Analysis and Design of Feedback Control Systems
 Fall 2004
 Quiz 1
 Wednesday October 6, 2004
OPEN BOOK

Problem 1 (35 pts)

A technique often used to improve response of robot arms is called "feed-forward" control. The objective is to get the arm to more exactly follow the joint angle command. A simplified version of this control system is shown below:



- What is overall transfer closed-loop transfer function $\frac{\theta(s)}{\theta_R(s)}$?
- How does the feed-forward loop change the characteristic equation of the system (compared to a system without $G_F(s)$)?
- Show that if $G_F(s) = 1/G_p(s)$ the robot will *perfectly* follow any command θ_R

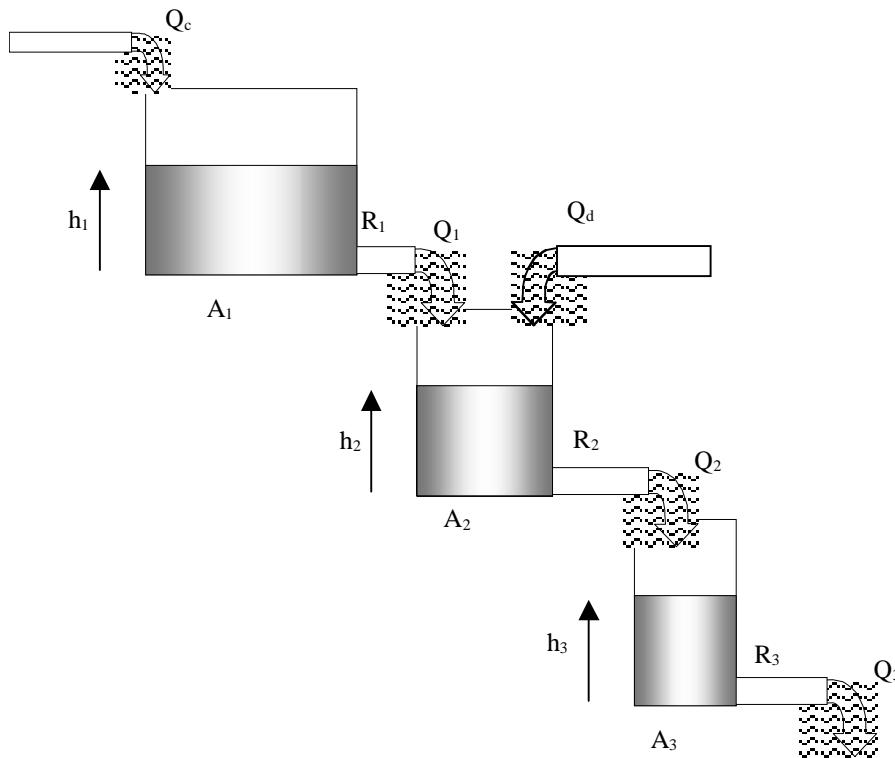
Now let's look at non-ideal conditions (i.e the *real world*).

Assume that $G_p(s) = \frac{1}{s+a}$ and $G_c = K_c$. In designing G_F the term "a" thought to be 1.0 but in reality has a value of 2. (In other words you used the "wrong" value for G_F .)

- How will the step response of the system change from the ideal? Please illustrate your answer with a sketch of the ideal and actual step response for two case $K_c = 1$ and $K_c = 10$.
- Can you overcome this mismatch by increasing the gain to larger and larger values?

Problem 2 (30%)

Multi-tank level control systems are typical in large refineries, municipal waste treatment systems and hydroelectric generating plants. A schematic of such a plant is shown below.



Each tank empties into the next, and there is no back flow (i.e. there is no flow back into the tank from the downstream side).

Each tank can be modeled as a simple first order system such that:

$$RA \frac{dq_{out}}{dt} + q_{out} = q_{in}$$

where R is the outlet resistance and A is the area of the tank. (Note that the outlet flow is proportional to the height, h , of water in the tank.)

a) Given this information, develop transfer functions for each tank, and assemble them into a block diagram. Please label the flows Q_c , Q_1 , Q_2 , Q_3 , Q_d , on the diagram.

Now assume that $R_1 = R_2 = R_3 = 1$, and $A_1 = 10, A_2 = 2, A_3 = 1$, We make a step change in the inlet flow Q_c and then measure the flow of each tank over time.

b) Please sketch on common axes, your estimate of what the Q_1, Q_2, Q_3 transients will look like. (Pay careful attention to the time constants of each tank.)

Now assume a flow controller of the form:

$$Q_c = K_c e = K_c (Q_r - Q_3)$$

where Q_r is the reference flow and K_c is the controller gain.

c) Draw the resulting block diagram for the entire closed-loop system.

d) Given the tank parameter from above, what is the closed – loop transfer function $\frac{Q_3}{Q_r}$ for this system?

e) How many of the closed-loop roots can we independently specify? (Please explain your answer.)

f) For this same system, what is the closed-loop disturbance transfer function $\frac{Q_3}{Q_d}$?

Problem 3 (35%)

In a high fidelity system (meaning one with a broad-band frequency response) speakers come in pairs, usually one for the low frequencies (the *woofer*) and one for the high frequencies (the *tweeter*). A test of the woofer has yielded the transfer function relating the input signal from the amplifier to the output speaker cone motion:

$$G_{\text{woofer}} = \frac{Y(s)}{V_{\text{in}}(s)} = \frac{500^2}{s^2 + 300s + 500^2}$$

- a) Sketch an *accurate* Bode diagram (gain and phase) on the attached graph paper. Start with an asymptotic plot and then add appropriate details to make it accurate.
- b) What is the resonant frequency , ω_r , of this system?

To avoid this resonance, a “low-pass” filter is placed on the electrical side of the system. The net effect is to attenuate the high frequencies in V_{in} . Assuming that the filter itself is given by:

$$G_{\text{lowpass}} = \frac{300^2}{s^2 + 600s + 300^2}$$

- c) What is the new Bode Diagram for the filter/speaker system? (You may add this plot to your sketch from part a)
- d) Has the filter eliminated the effect of the speaker resonance? Please explain by use of any appropriate magnitude plot on the Bode diagram

Since the cutoff point of the filter is well within the useful range of audio (which is typically 10 - 15,000 Hz or 62.8 to 94,000 rad/sec), we need another speaker to cover the high range. For this we use a smaller cone (since high frequencies don't require as much air movement as low frequencies) and a correspondingly smaller voice coil. All this adds up to a speaker with the following input - output equation:

$$G_{\text{tweeter}} = \frac{120,000^2}{s^2 + 24,000s + 120,000^2}$$

- e) Is a “low pass” filter needed here to avoid exciting the tweeter resonance? Please explain your answer *carefully*.

