2.14 Quiz 1 2004 Solution

Problem 1

1) $\frac{\theta}{\theta_R} =$; a) $\theta = G_p\,(G_c(s)(\theta_R - \theta) + G_F\,\theta_R$

$$\theta\,(1 + G_c G_p) = (G_p G_c + G_p G_F)\,\theta_R$$

$$\frac{\theta}{\theta_R} = \frac{G_p\,(G_c + G_F)}{1 + G_c G_p} \quad //$$

b) G_F does not appear in the denominator ∴ it is not part of the C.E.

$$CE \Rightarrow 1 + G_c G_p = 0 \approx \text{loop C.E.}$$

c) If $G_F = \frac{1}{G_p} \Rightarrow \frac{\theta}{\theta_R} = \frac{G_c G_p + G_p/G_p}{1 + G_c G_p} = 1$ "ideal"

d) $G_p = \frac{1}{s+2}$; $G_c = \left(\frac{s+1}{1}\right)$ $K_c = G_c$

$$\Rightarrow \frac{\theta}{\theta_R} = \frac{\left(\frac{1}{s+2}\right)\left(K_c + \frac{s+1}{1}\right)}{1 + K_c/s+2}\,\frac{(s+2)}{(s+2)} = \frac{K_c + s + 1}{s + (2 + K_c)}$$

$$\frac{\theta}{\theta_R} = \frac{s + (1 + K_c)}{s + (2 + K_c)} \leftarrow 1^{\circ} \text{ with a zero in "I"}$$

Alternative answer to (b)

$$\frac{\theta}{\theta_R} = \frac{(G_c + G_f)G_p}{1 + G_c G_p}$$

→ The C.E. is the characteristic equation of the overall differential equation for the system. As such it should represent the dynamics when the input $(\theta_R) = 0$

With this argument G_f is no longer relevant so we revert to

$$\frac{\theta}{\theta_R} = \frac{G_c G_p}{1 + G_c G_p} \quad \& \text{ C.E. } 1 + G_c G_p$$

→ If we look at the overall T.F and consider the form:

$$\frac{\theta}{\theta_R} = \frac{\left(\frac{N_c}{D_c} + \frac{N_F}{D_F}\right)\frac{N_p}{D_p}}{1 + \frac{N_c N_p}{D_c D_p}}$$

and factor, the we get:

$$\frac{D_c D_p \left(\frac{N_c N_p}{D_c D_p} + \frac{N_F N_p}{D_F D_p}\right)}{D_c D_p + N_c N_p}$$

$$= \frac{N_c N_p + \frac{N_F N_p}{D_F}}{(\qquad)} \frac{D_F}{D_F} = \frac{D_F N_c N_p + N_F N_p}{D_F D_c D_p + N_c N_p D_F}$$

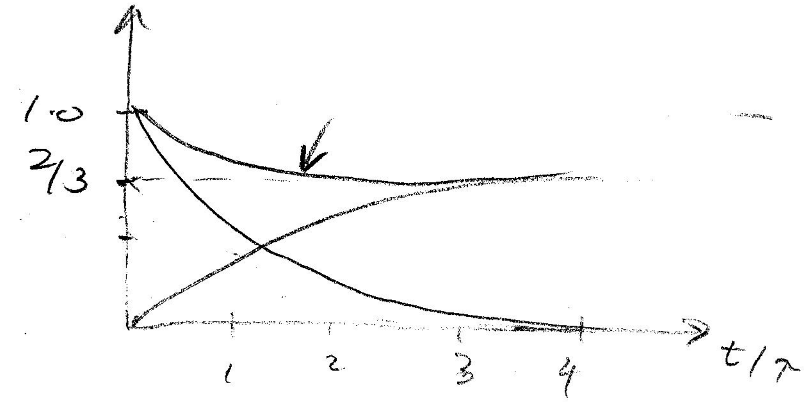
So it appears that the C.E. $= D_F (D_c D_p + N_c N_p) = 0$

Either answer will be accepted

If $K_c = 1$ $\quad \frac{\Theta}{\Theta_R} = \frac{s+2}{s+3}$

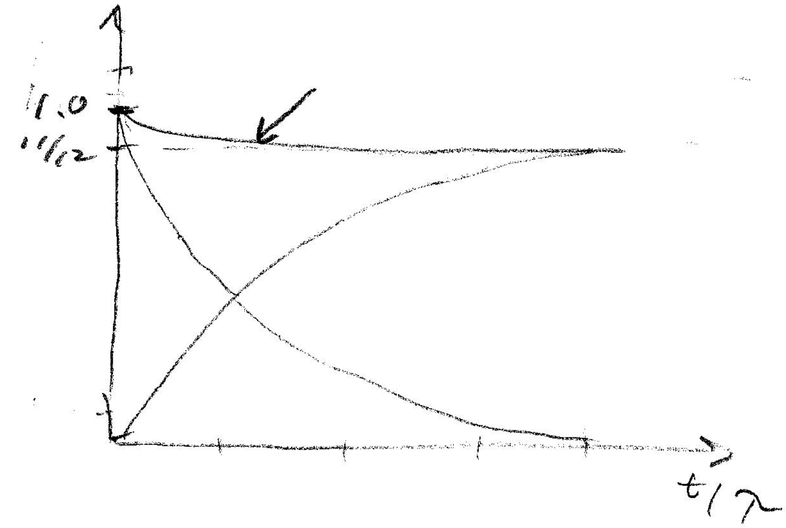
$\Rightarrow \quad \dot{y} + 2y = \dot{u} + 3u$

1° with $\tau = 1/3$
Step magnitude $= 2/3$
Impulse magnitude $= 1$ [IVT → $\frac{s}{s} = 1$]



If $K_c = 10$ $\quad \frac{\Theta}{\Theta_R} = \frac{s+11}{s+12}$

$\tau = 1/12$ $\quad$ Step $= 11/12$ $\quad$ Impulse $= 1$



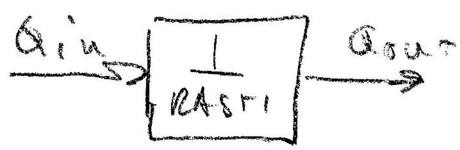
c) Very large $K_c \to \tau \to 0$ and the impulse magnitude approaches the final value → toward 1.0 or "ideal"

Problem 2

a) Each tank is described by:

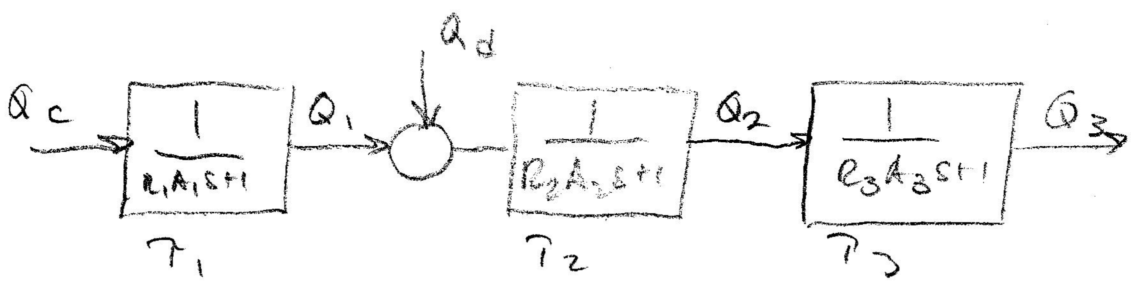
$$RAsQ_{OUT} + Q_{OUT} = Q_{in}$$

$\Rightarrow$



$\tau = RA$

Thus for the system we get



b)

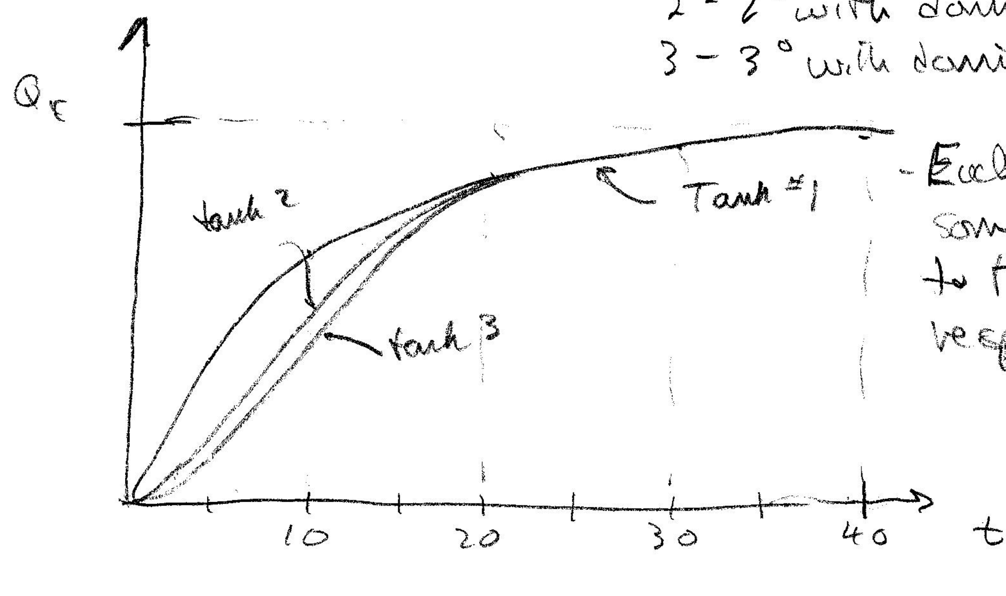
$\tau_1 = 10$ $\qquad$ $\tau_2 = 2$ $\qquad$ $\tau_3 = 1$

longest, dominant

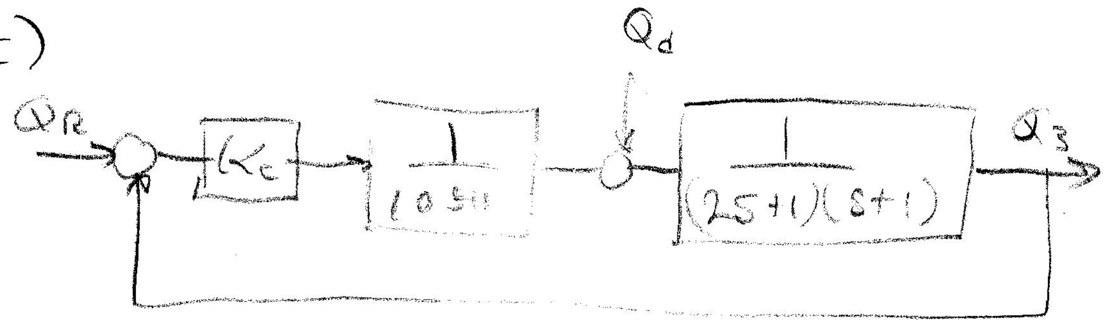
1 - simple step w $\tau = 10$ $\therefore t_s/_{2\%} \cong 40$

2 - 2° with dominant $\tau = 10$

3 - 3° with dominant $\tau = 10$



- Each tank adds some (smaller) lag to the initial response c)



d) $$\frac{Q_3}{Q_R} = \frac{K_c}{(10s+1)(2s+1)(s+1)+K_c} \qquad \left[\frac{\frac{K_c}{(\ )(\ )}}{1+\frac{K}{(\ )(\ )(\ )}}\right]$$

$$= \frac{K_c}{12s^3+32s^2+13s+(1+K)}$$

e) There are 3 roots but only one variable coefficient ∴ we can only specify one root independently

f) $$\frac{Q_3}{Q_d} = \frac{\frac{1}{(2s+1)(s+1)}}{1+\frac{K}{(\ )(\ )(\ )}} = \boxed{\frac{10s+1}{12s^3+32s^2+13s+(1+K)}}$$

Problem 3 (35 pts)

(a) 10 pts, see plot

(b) 5 pts, $\boxed{\omega_r = \omega_n \sqrt{1 - 2\zeta^2} \approx 453 \frac{rad}{s}}$ ← from Lecture #6, Sep. 27... on back cover of Nise, this is "ω_p" under "Freq. Resp."

(c) 10 pts, see plot

(d) 5 pts, $\boxed{\text{yes, resonance is eliminated}}$ – see plot...

- At $\omega_n = 500 \frac{rad}{s}$, magnitude is $20\log_{10}\left(\frac{1}{2\zeta}\right) \approx 4.4\,dB$ ← [for $G_{woofer}(s)$ alone]. It is somewhat higher at ω_r. From the plot on p. 615 of Nise for $\zeta = .3$, peak is perhaps 5dB. (If you want an exact peak magnitude:

$$\left| G_{Woof}(453) \right| = \frac{500^2}{\sqrt{(500^2 - 453^2) + (300 \cdot 453)^2}} \approx 4.8\,dB,$$

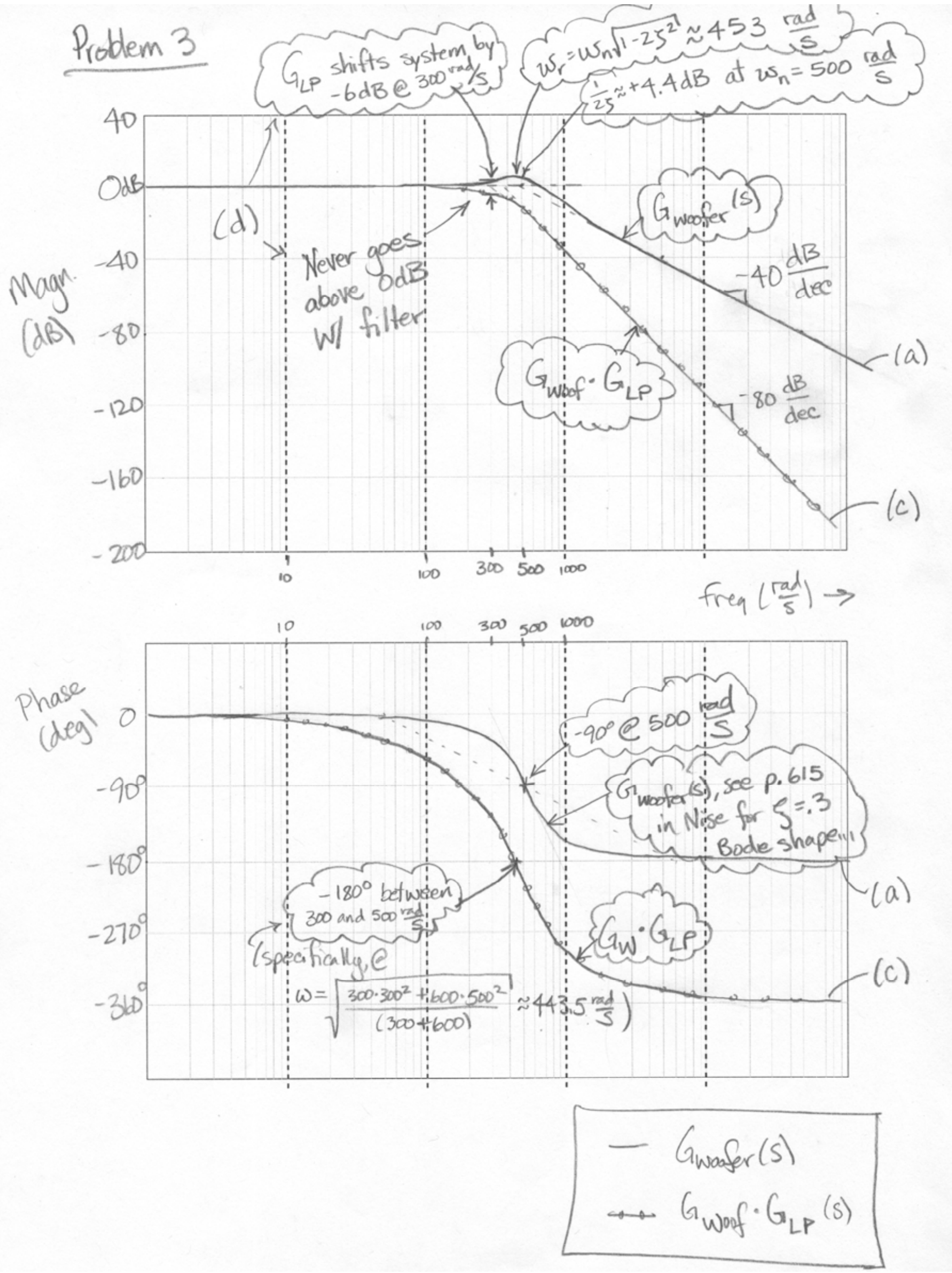
but look at plot or saying "close to 4.4 dB" is good enough here...)

- For $G_{LP}(s)$, magnitude is already $-6\,dB$ @ $300 \frac{rad}{s}$ (ω_n for LP filter is $300 \frac{rad}{s}$, $\zeta_{LP} = 1 \rightarrow 20\log_{10}\left(\frac{1}{2}\right) \approx -6\,dB$), & $|G_{LP}(\omega = 453)|$ is clearly even lower -- so we push the combined $G_{woofer} \cdot G_{LP}$ system below 0dB. (see plot!)

(e) 5 pts, $\boxed{\text{no, this resonance is above human hearing.}}$

$\left(120{,}000 \frac{rad}{s} > 94{,}000 \frac{rad}{s}\right)$

Problem 3



G_LP shifts system by -6dB @ 300 rad/s
ω_r = ω_n √(1-2ζ²) ≈ 453 rad/s
1/(2ζ) ≈ +4.4dB at ω_n = 500 rad/s
Magn. (dB)
40
0dB
-40
-80
-120
-160
-200
(d)
Never goes above 0dB w/ filter
G_woofer(s)
-40 dB/dec
(a)
G_woof · G_LP
-80 dB/dec
(c)
10
100
300
500
1000
Freq (rad/s) →
Phase (deg)
0
-90°
-180°
-270°
-360°
-90° @ 500 rad/s
G_woofer(s), see p. 615 in Nise for ζ=.3 Bode shape...
(a)
-180° between 300 and 500 rad/s
(specifically @ ω = √((300·300² + 600·500²)/(300+600)) ≈ 443.5 rad/s)
G_W · G_LP
(c)
G_woofer(s)
G_woof · G_LP (s)