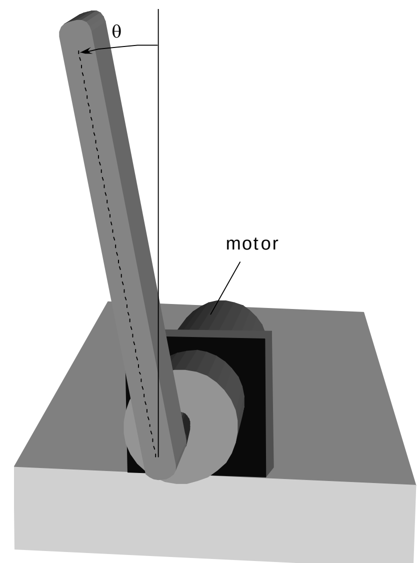
<u>OPEN BOOK</u>

<u>Problem 1 (30 pts)</u>



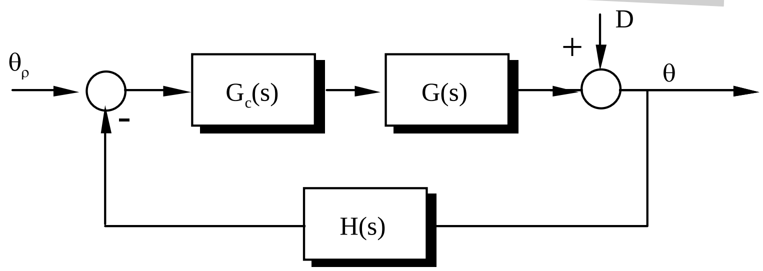
In this problem, we want to create a control system to keep a pendulum in an upright position. The overall design goals are:

- settling time after a step command or disturbance of 1 sec
- zero overshoot.

For small angles of operation (θ=± 10º), the inverted pendulum system (shown right) can be modeled as a spring-mass-damper system with a negative (unstable) spring.

Assume the transfer function relating motor current to pendulum angle is given by: $G(s) = \frac{10}{(s-1)(s+2)}$

a) Plot the root locus for $G_c(s) = K_c$ and H(s) = 1



It has been suggested that the angular velocity ($\dot{\theta}$) of the pendulum be measured and fed back in addition to the angle to help in the controlling the system.

b) Sketch a new block diagram to show how this measurement would be incorporated.

c) Using this velocity feedback along with the proportional controller, sketch a root locus that shows how you can achieve the design specifications (i.e. no overshoot, settling time about 1 sec. Slower response is not acceptable, but too fast/"jerky" is not desired either.)

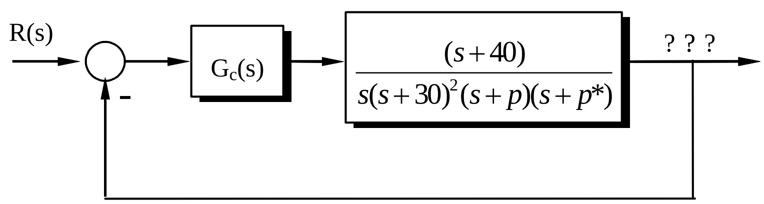
d) Calculate K_c and *H(s)* corresponding to your design in part (c), and calculate the steady-state angular error (percent error) for a step command in angle. (You should use your root locus in calculating the required gain. Be careful to consider that gains in G(s) and H(s) may already contribute to the root locus "gain", even when Kc=1.)

e) It has been suggested that we add acceleration feedback to the system to improve performance. Assuming that this would add an additional <u>real</u> zero to the feedback path, determine if this would have any benefits on either response time or error again by reference to the root locus that would result.

Problem 2 (30 pts)

The high-speed transport of magnetic tape is a critical problem in archiving of digital data. Such systems are required to speed up quickly and then maintain a desired speed before stopping at a precise location. The block diagram of such a system is shown below.

It is desired to have a system with zero steady state error to a step, and velocity error of 0.2%. The transient response should be such that the 2% settling time is about 0.8 sec.



p = 20 + j20   p* = 20 – j20

For this system,

a) Sketch the root locus. Do be precise in plotting asymptote locations, BUT use only approximate sketching methods for breakaway points and angles of departure. (Do not waste time calculating any breakaways *too* exactly!)

b) Confirm that only gain compensation is necessary to achieve the settling time specifications. Be specific in your explanation by tying it to the root locus.

c) Confirm that the gain compensated system will not meet the velocity error specifications by determining (approximately) the velocity constant (K_v) of the system from the root locus. (If you have trouble finding this gain, assume a value of $K_V = 1$ and move on to part d.).

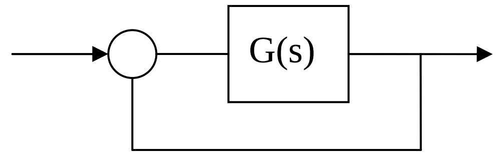
d) Design a lag compensator to meet the velocity error.

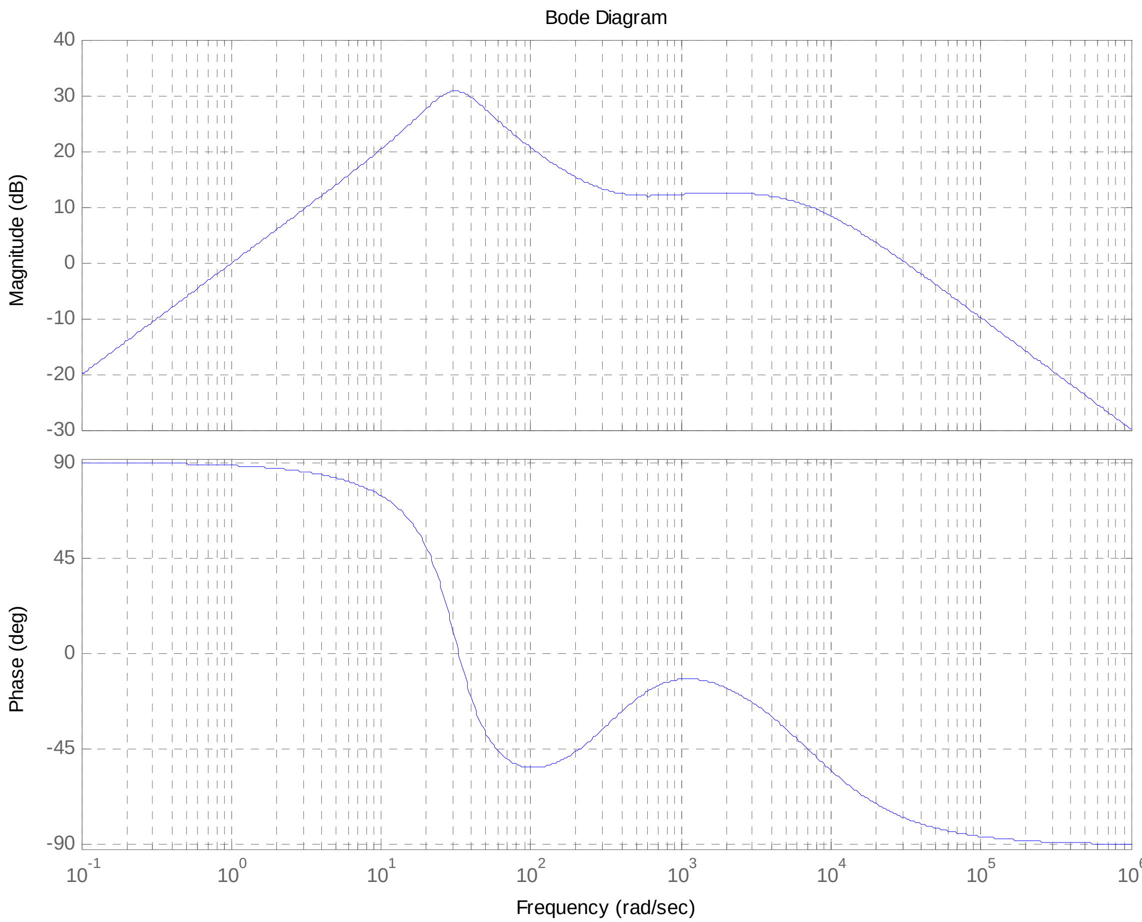
Please note that it is not necessary to plot a Bode Diagram to design the lag compensator, but you must be careful not to disrupt the crossover frequency.

Problem 3 (25 pts)

The Bode diagram below shows the frequency response for the following system with unity feedback (H=1):

$$G(s) = \frac{32000s(s+420)^2}{(s+840)(s+7000)(s^2+28s+980)}$$





a) Sketch the approximate Bode plot (**magnitude only**) for the closed-loop system (|T|) $T(s) = \dfrac{G(s)}{1+G(s)H(s)}$. (H=1) *You may sketch T(s) directly onto the plot above, but be sure to turn in the plot with your quiz if you do!!*

b) What is the **bandwidth** of the closed-loop system, T(s)?

c) For what range of frequencies will we **have less than 10% error** between the amplitude of the commanded input and amplitude of the resulting output of T(s)?

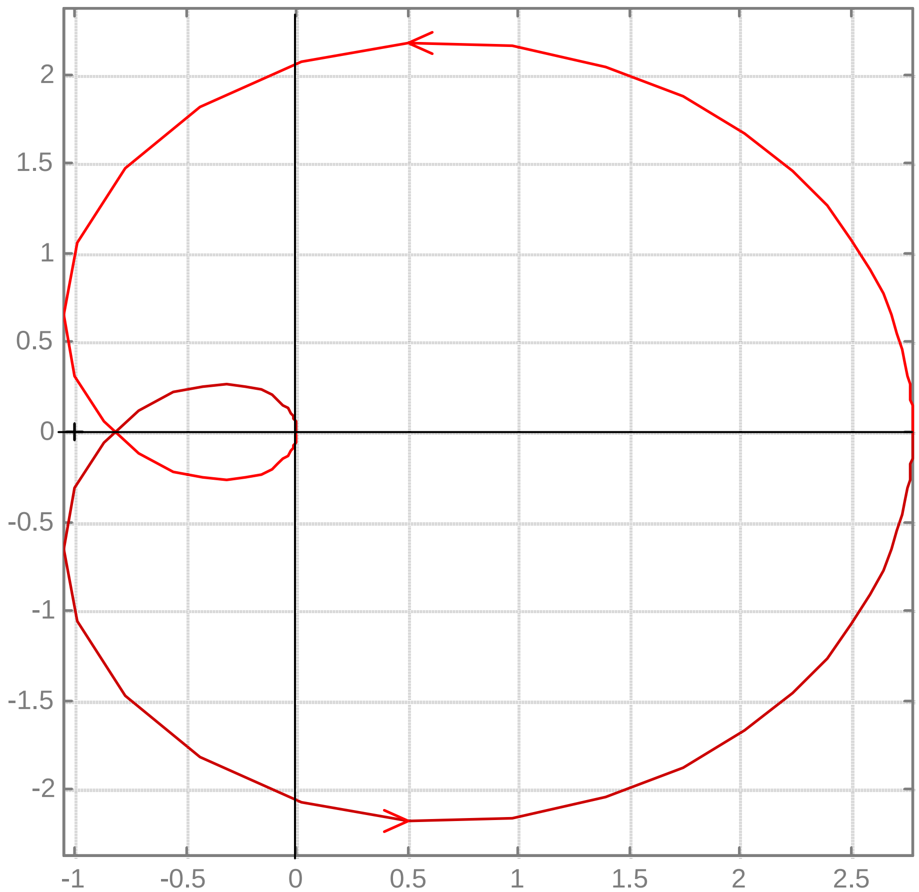
d) If you were given just the |T| plot that you have sketched, what transfer function would you assume it came from?

Problem 4 (15 pts)

Consider the following open loop transfer function:

$$G(s) = \frac{K(s+5)}{s^2 - 6s + 9}$$

The corresponding Nyquist diagram is given by:



At the gain chosen to create this plot,

a) Will the closed-loop system $T(s) = \dfrac{G(s)}{1+G(s)}$ be stable or unstable? Please explain in terms of the Nyquist stability criterion.

b) What will happen as the gain K is increased? (Again, justify your answer via Nyquist.)

NB: You might find it useful to make a quick sketch of the root locus for this system.