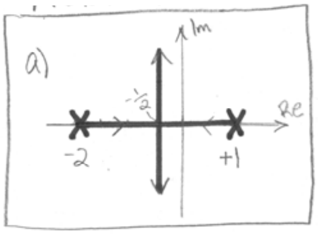
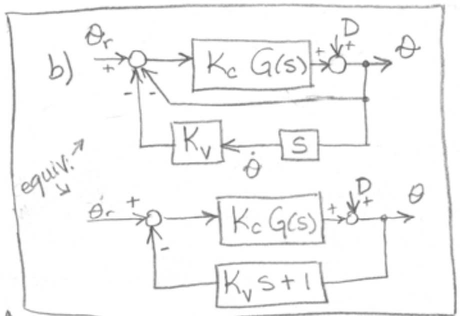
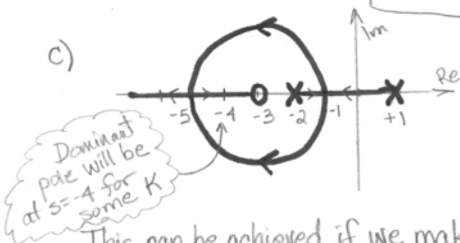
2.14 Quiz 2 Solutions

Problem 1





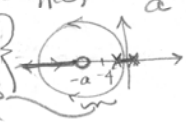
c)



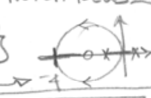
No overshoot & $T_s = 1$ sec, so want real pole at $s=-4$. 2^{nd} pole needs to be faster than this (i.e. further left on real than $s=-4$).

This can be achieved if we make $H(s) = \frac{s+a}{a}$, where

$2\frac{4}{7} \le a < 4$ (If $a > 4$, then $s=-4$ NOT on locus.)



(If $a = 2\frac{4}{7}$, locus breaks in @ $s=-4$)



d) One choice is: $H(s) = \frac{s+3}{3}$

Then $GH = K_c \frac{10}{3} \frac{(s+3)}{(s-1)(s+2)}$; to get pole @ -4 → $K_c \frac{10}{3} = \frac{(5)*(2)}{(1)} = 10$

($|-4-1|$, $|-4+2|$, $|-4+a|$)

So $K_c = 3$ puts slower pole at $s=-4$.

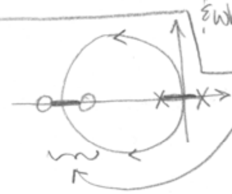
$$T(s=0) = \frac{G(s=0)}{1+G(s=0)H(s=0)} = \frac{\left(\frac{10K_c}{-2}\right)}{1+\left(\frac{10K_c}{-2}\right)(1)} = \frac{-15}{1-15} = \frac{15}{14}$$

$e_{ss} = 1 - T(s=0) = \frac{1}{14}$

more Generally: $e_{ss} = \frac{5K_c\left(1-\frac{1}{H(0)}\right)+\frac{1}{H(0)}}{5K_c - \frac{1}{H_0}}$

& when $H(0)=1$: $e_{ss} = \frac{1}{5K_c - 1}$

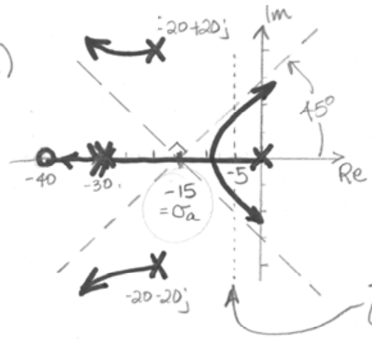
e)



2^{nd} real zero just restricts how fast the faster pole can be. Can make single zero very close to $s=-4$ to make $|-4+a|$ tiny & thus make K_c large & e_{ss} ↓. Do not need 2^{nd} zero to do this.

<u>Problem 2</u>

a)



b) $T_s \approx 0.8s$, so want real part of slowest pole (or pole pair) at $\boxed{s = \frac{-4}{.8} = -5}$

Two choices possible on locus:

I) single real pole @ $s = -5$

II) pole pair @ $s = -5 \pm 10j$

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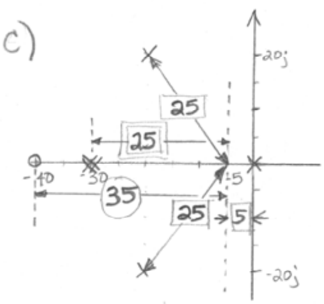
c)



First, calculate K_c that puts one real pole @ $s=-5$ (option I):

$$K_{c_I} = \frac{\Pi\,(\text{dist's to OL poles})}{\Pi\,(\text{dist's to OL zeros})} = \frac{(25)^4(5)}{35}$$

$K_{c_I} \cong 55,800$ (for II, $K_{c_{II}} \approx 135,000$)

Then, $K_v = K_c\,(sG(s))\big|_{s=0} = K_c \frac{(40)}{(900)(800)}$

$\boxed{K_{v_I} = 3.1}$, but we <u>want</u> $K_v > 500$ (for vel. err $< \frac{1}{500}$)

So we need to increase $(sG(s))\big|_{s=0}$ by a factor $\frac{500}{3.1} \approx 160$

if we chose <u>$s=-5$</u> (single pole option I), OR

(if we chose <u>$s=-5\pm 10j$</u>, then $K_{v_{II}} \approx 7.5$, need to increase by 67)

d) $\boxed{\begin{array}{l} \alpha_I = 160 \\ \omega_{c_I} \approx 5 \frac{rad}{s} \\ \frac{1}{T_I} = \frac{\omega_c}{10} \\ \frac{1}{T_I} = .5 \end{array}}$ or $\boxed{\begin{array}{l} \alpha_{II} = 67 \\ \omega_{c_{II}} \approx \sqrt{5^2+10^2} \\ \quad \approx 11 \frac{rad}{s} \\ \frac{1}{T_{II}} = \frac{\omega_c}{10} \\ \frac{1}{T_{II}} = 1 \end{array}}$

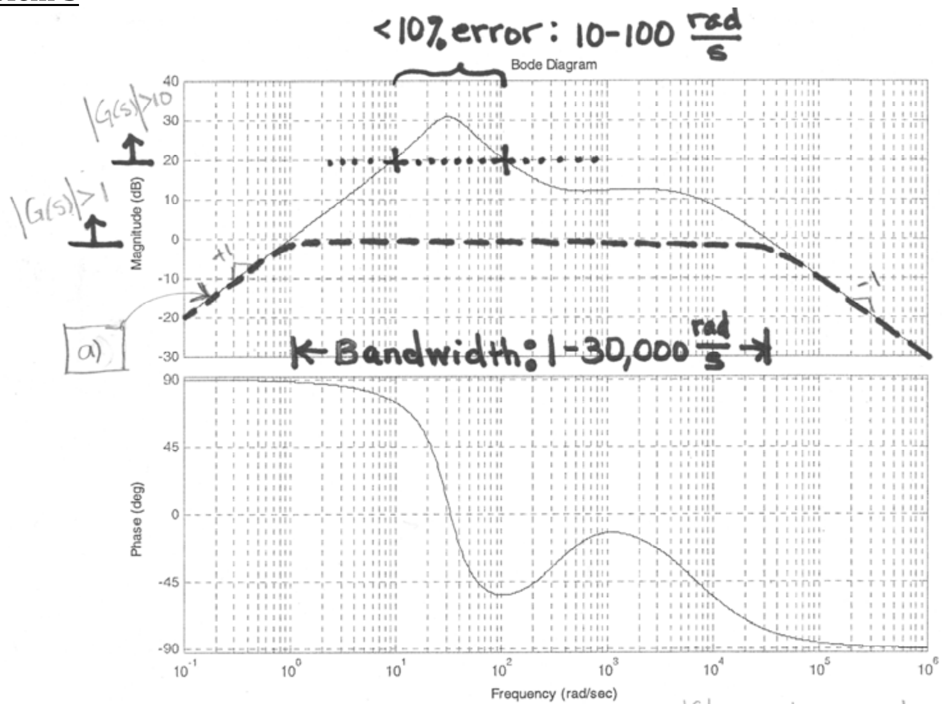
$$lag = \frac{s + \frac{1}{T}}{s + \frac{1}{\alpha T}}$$

← pick zero <u>about</u> 1 decade before dominant pole(s)

$$lag_I = \frac{s+.5}{s+.003}$$

$$lag_{II} = \frac{s+1}{s+.015}$$

Problem 3



a) Where $|G(s)|>1$, $|T(s)| \approx 1 - \frac{1}{|G(s)|} \approx 1$, $\left[|T| \approx \frac{|G|}{1+|G|} \approx \frac{1}{\frac{1}{|G|}+1} \approx 1 - \frac{1}{|G|} \approx 1\right]$

Where $|G(s)|<1$, $|T(s)| \approx |G(s)|$

b) Where $|G(s)|>1$ → from $1\ \frac{rad}{s}$ to $30{,}000\ \frac{rad}{s}$ is the bandwidth

c) Where $|G(s)|>10$ (or $|G(s)|>9$) → from 10 to 100 $\frac{rad}{s}$ error is < 10%

d) $T(s) \approx \frac{30{,}000\,s}{(s+1)(s+30{,}000)}$

Problem 4

a) No encirclement of −1, N=0

2 poles in RHP in O.L.T.F., P=2

Z = P − N = 2 − 0 = 2 (see p. 623)

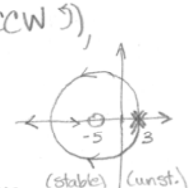
unstable poles in C.L.T.F. is 2,

∴ T(s) is unstable

b) As K↑, eventually N=2 (CCW ↺),

so Z = P − N = 2 − 2 = 0,

T(s) becomes stable



(Both poles in RHP for "small" K, but move together into LHP as K↑ in root locus...)