Problem 1:

For the following circuits:

- Construct the appropriate reduced bond graphs
- Show causality on all bonds
- Derive the state equations and express them in the form \( \dot{x} = Ax + Bu \)

a) 2\textsuperscript{nd} Order Low Pass Filter

b) Differentiator

c) Phase Lag Filter

d) Bridged-T Circuit
Problem 2

The system shown below is part of typical power steering pumps and the goal is to provide a constant flow regardless of variations in load pressure or supply flow. There is also a relief valve built into the spool valve to avoid excessive load pressures. The pressure difference across the variable control orifice causes the spool valve to move against the preloaded spring $k_v$. This in turn varies the size of the opening in the valve that discharges to the supply tank.

- Identify the ideal elements to be used to model this system
- Create a bond graph, and include causality
- Derive the state equations and express them in the form $\dot{x} = Ax + Bu$
Problem 3

The two systems shown below each have two tanks fed by a flow source and the outlet of each tank has a value or flow restriction (R). However, the two are significantly different.

To illustrate these differences:

a) Derive a bond graph for each, explaining your assumptions along way.
b) Derive the state equations and comment on how the system matrices A and B are different or similar.
c) Explain heuristically why one system can never oscillate while the other can for some set of parameters (tank areas A and resistances R).
d) Can you relate this heuristic to the differences in the A matrices between the two?

Hint: you might find the concept of an “active bond” useful for this problem. An active bond: \( A \xrightarrow{e} B \) implies that system A imposes an effort “e” on B, but there is no corresponding flow “f”. The systems are in effect “decoupled”.

![Diagram of systems](image-url)
Problem 4

A classic system dynamics problem is that of vehicle suspension. The 2-D problem can be posed as shown below:

In this schematic, the forces $F_1$ and $F_2$ represent the forces on the front and rear wheels respectively, and the springs and dampers the wheel suspension system. The masses $m_1$ and $m_2$ represent the wheel & tire mass and the vehicle body is represented by an object with mass $M$ and rotational inertia $J$. Clearly the body mass can both translate up and down and rotate.

The goal here is to derive a set of state equations for this system using bond graphs. However, the body mass present a new twist. We have an object that has two power ports (one from each of the suspension system) and has two degrees of freedom (translation and rotation)

The following bond graph has been proposed to model this situation

a) Add causality to this bond graph and confirm that veracity of this bond graph by deriving the equations of for the body mass given the force inputs from the wheels. (Why is this model limited to small rotations of the body mass?)

b) Draw the entire bond graph for this system including the inputs and wheel suspension systems.

c) Derive the state equations for the complete system.

Now consider the problem of creating a better model of the road – tire interface. Roads are best described not as force sources but as displacement or velocity sources.
d) How would you augment the model and the bond graph to allow the inputs to be velocity sources. What new physical elements would you be including the model and how are they incorporated?

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\begin{align*}
\text{TF: } & \frac{L}{2} \\
\begin{array}{c}
\dot{m} \\
\dot{J} \\
J \\
\end{array} & \begin{array}{c}
1 \\
1 \\
1 \\
\end{array} \\
\begin{array}{c}
0 \\
F_1 \\
F_2 \\
\end{array} & \begin{array}{c}
v_1 \\
\tau_1 \\
v_2 \\
\end{array} \\
\end{align*}
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