Problem 1:

Consider the simple relay shown schematically below. It is activated by electrical power and the desired result is to have the relay arm snap to full contact with the stop as shown.

For this system:

a) Develop a bond graph

b) Now consider the problem of flux leakage across the circuit as indicated below. How would you model this effect? Augment your bond graph from part a) to show this change.

c) Using the model from part b, derive the state equations for the relay.

d) What is an equivalent purely electrical system for the relay as viewed from the input port?

e) It is well known that devices such as these get hot after continuous operation. How would you account for this dissipation in the bond graph?
Problem 2

Given the following A matrices:

\[
A = \begin{bmatrix}
4 & 1 & 1 \\
1 & 4 & 1 \\
1 & 1 & 4
\end{bmatrix}
\]

a) Find the resolvent using manual matrix methods
b) Find the corresponding state transition matrix
c) Solve for the homogeneous response of the system to \(x_1(0) = x_2(0) = 1; x_3(0) = 0\)

Problem 3

For the following systems:

\[
\begin{align*}
A &= \begin{bmatrix}
1 & 0 & 0 \\
1 & 2 & 0 \\
1 & 2 & 3
\end{bmatrix} & B &= \begin{bmatrix}
1 \\
1 \\
1
\end{bmatrix} & C &= \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 1 & 0
\end{bmatrix} & D &= 0 \\
A &= \begin{bmatrix}
0 & 1 & 1 \\
0 & 1 & 1
\end{bmatrix} & B &= \begin{bmatrix}
1 \\
1 \\
1
\end{bmatrix} & C &= \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 1 & 0
\end{bmatrix} & D &= \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0
\end{bmatrix}
\end{align*}
\]

Draw a block diagram of these systems relating all inputs to all outputs, using the time integral block:

![Block diagram](image)

along with appropriate gains and summing junctions.
Problem 4

The system shown below is a rough schematic of a fluid transmission. The input is a torque source and it sees a load that looks like an angular velocity source.

For this system:

a) Create a bond graph model
b) Derive the state equations and express them in the form $\dot{x} = Ax + Bu$
c) Write an output equation $y = Cx + Du$ such that the states $x$ are the outputs.

With both the Torque and Velocity sources as impendent inputs, and the outputs from part c):

d) Determine the transfer function matrix for this system (You may use MATLAB or do it manually)

e) Comment on this matrix with respect to
   - The expected characteristic response
   - The effect of the numerator terms on the step response
   - Based on each of the four entries in this matrix, make a prediction of
     the step response of each input – output pair

f) Now look at the computed response using the MATLAB “STEP” function:
   e.g. step(A,B,C,D)
   How does the result compare to your expectations

Please explain in detail.