Problem 1

We have the problem of controlling a simple robot arm, but one that is coupled to the actuator by an elastic drive belt. A schematic model of this system is shown below.

The state equations are given by:

\[
\begin{bmatrix}
\dot{q}_1 \\
\dot{q}_2 \\
\dot{\dot{q}}_1 \\
\dot{\dot{q}}_2
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 & 0 \\
-\left(\frac{k}{J_1}\right) & \left(\frac{b_1}{J_1}\right) & \left(\frac{k}{J_1}\right) & 0 \\
0 & 0 & 0 & 1 \\
-\left(\frac{k}{J_2}\right) & \left(\frac{b_2}{J_2}\right) & \left(\frac{k}{J_2}\right) & 0
\end{bmatrix}
\begin{bmatrix}
q_1 \\
q_2 \\
\dot{q}_1 \\
\dot{q}_2
\end{bmatrix} + 
\begin{bmatrix}
0 \\
0 \\
1/J_1 \\
0
\end{bmatrix}
t_m
+ 
\begin{bmatrix}
0 \\
0 \\
0 \\
1/J_2
\end{bmatrix}
t_d
\]

where \(t_m\) is the motor torque and \(t_d\) is a disturbance torque.

a) With parameters given below determine the corresponding transfer function matrix for the system

\[
J_1 = 10 \\
k/J_1 = 16 \\
k/J_2 = 9 \\
b_1/J_1 = 2 \\
b_2/J_2 = 1 \\
l/J_2 = 0.2.
\]

b) Prove by example that if all states are fed back through independent gains, all of the eigenvalues of the system can be determined independently by varying the four controller gains. (assume the controller \(u=[k_1 k_2 k_3 k_4] x\))
Given that the goal is to control the output position $q_2$, 

c) Derive the characteristic equation of the system for the case where one term involves
the error $q_{REF} - q_2$. This implies that $u = k_1 x_1 + k_2 x_2 + k_3 (x_{ref} - x_3) + k_4 x_4$

where $x_{ref}$ is $q_{REF}$

We want to design a controller to meet the specifications:
  - A step response settling time of 1.0 sec
  - A maximum overshoot of 5%
  - Rejection of disturbances ($t_d$) in the frequency range $1 < \omega_d < 10$ rad/sec
    - If possible minimize the actuator power required.

For this system:

d) Design a controller using pole placement techniques. For this design use two different
methods to get the desired poles:
   - Using Root locus or Bode Diagram approximations
   - Using the Butterworth pole discussed in class

f) Compare the controller effort for the two designs as well as conformance to the design
specs. You can define the controller effort as the square of the controller output ($u$ or $\tau_m$)
over a specified interval of time

g) Once you have designed the controller, how would you evaluate the response to the
disturbance $t_d$?

h) Given the closed-loop controller, can you write a set of state equations that will allow
the system to have reference input in the position $q_2$?

Problem 2

Now look at the two closed-loop systems designed in Problem 1.

Using singular value decomposition (See matlab function SVD) compare the two designs
with respect to:

- minimum bandwidth
- frequency band for noise to be rejected to 1%
- frequency band over which the disturbance will be attenuated by at least 10%

Problem 3

Do Problem 6.7 in Friedland. Notice that this is a case where the system actually has two
control inputs ($\tau_R$ and $\tau_A$), but you are given a linear equation relating these and the
states. This reduces the problem to a single input case where the pole placement
technique can be used.