Problem 1 (40 %)

The figure shown below is for a reciprocating pump driven by 60Hz line voltage. It is a schematic of a typical aquarium pump used to aerate a water tank. We want to develop a model of this pump using bond graphs so that we can eventually estimate the pump efficiency.

The pump operates as follows:

The 60 Hz line voltage excites the coil, producing a 60 Hz mechanical input to the lever via interaction with the magnet. The pump will operate most efficiently if the natural frequency of the mechanical system is at or near 60Hz.

As the lever moves up and down under the influence of this input, the rubber bellows expand and contract. Air is admitted to the bellows chamber through the left hand check valve on the upstroke and expelled through the right hand check valve on the down stroke.

For your model, you can make the following assumptions:

- The coil magnet interaction is exactly the same as that found in a permanent magnet DC motor
- The air is incompressible
- The lever is of length L and the bellows is attached at mid-length
Each check valve has the following characteristic -> where $e$ is the threshold pressure at which the valve opens.

From this plot it is clear that only one valve can be open at any time.

For this system:

a) Define the lumped elements of this system, and justify your definitions for each

b) Construct a bond graph for your model

c) Add causality and see if any conflicts occur. If so, comment on why.

d) Will your model be useful in predicting the resonant frequency of the system?

e) If not, suggest modifications to the model that would help it do so, and draw an new bond graph to confirm the improvement

f) What if the air were considered compressible. How would the model change (Do not redo the model just describe the difference)

Problem 2: (20%)

The figure below shows a design for a “shock –free” toboggan. We want to develop a model and set of state equations to see just how good it is.

a) Designate the lumped elements and their location on a sketch of the sled

b) Develop a bond graph for this system

c) Augment the bond graph fully and derive the state equations

d) Develop an output equation to determine the force on the passenger compartment and the force on the bottom of the toboggan.
Problem 3 (30 \%) 

For this problem we return to Deer Island outfall problem. Now we have been asked to develop a model of the multiple risers as shown in the figure below.
First, consider just a single riser. Since it is a long pipe, we must consider the inertia of the fluid in this pipe as well as the flow resistance. For simplicity, consider only the pipe flow resistance and not that of the diffuser head.

For the single riser system:

a) Sketch a physical model of the system you are considering and then the appropriate bond graph

b) Augment the bond graph and check causality

c) Make any adjustments to the model necessary to avoid any dependent elements, and sketch the new bond. Note any equivalent element parameter values.

Now consider a system with 2 (or more) risers.

d) Starting with your final bond graph from above, show how additional risers would be added, but sketching a bond graph for a 2 rise system

e) Show how additional risers would be added so that a full 55-riser model could result.