Massachusetts Institute of Technology
Department of Mechanical Engineering

2.160 Identification, Estimation, and Learning

Spring 2006

Problem Set No. 1

Out: February 13, 2006 Due: February 22, 2006

Problem 1

Consider the following example of least squares estimation. Given

$$3\theta_1 + 2\theta_2 = 6$$
$$\theta_1 + \theta_2 = 1$$
$$4\theta_1 - 6\theta_2 = -3$$
$$2\theta_1 + \theta_2 = 4$$

Obtain the least squares estimate of parameters θ_1, θ_2. Suppose an additional measurement:

$$2\theta_1 + 2\theta_2 = 3$$

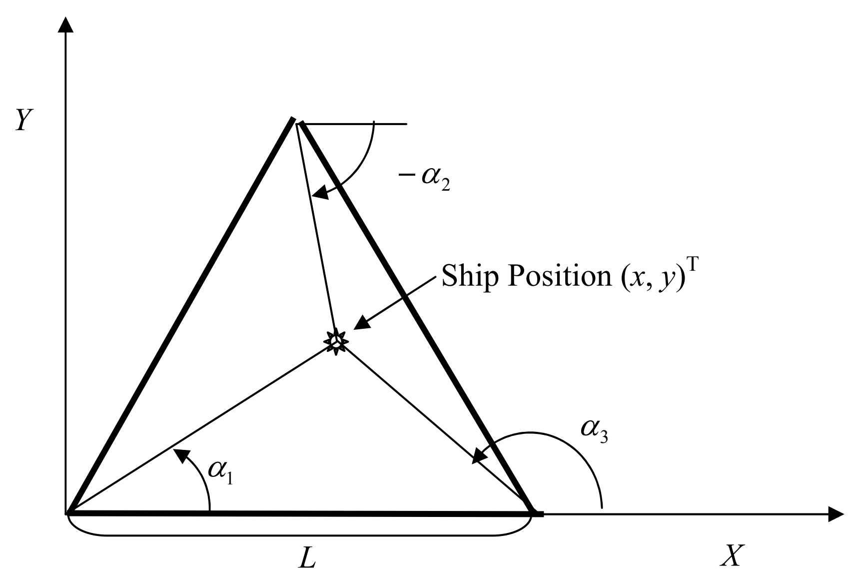
is obtained. Find the revised estimate by two methods:

Directly, by batch processing

Indirectly, by the recursive algorithm.

Problem 2

Shown below is a calibration range for determining the position of a ship. Three theodolite stations are placed at the three apexes of an equilateral triangle. Formulate the procedure for determining an optimal estimate of the ship's location, coordinates x and y, based on three angular measurements, $\alpha_1, \alpha_2, \alpha_3$. First, obtain a model equation where unknown parameters are ***linearly*** involved, and then give an optimal estimate. Write out the elements of the matrices involved.



Problem 3

Consider the recursive least-squares algorithm with exponential weighting:

$$\hat{\theta}(t) = \hat{\theta}(t-1) + \frac{P_{t-1}\varphi(t)}{\alpha + \varphi^T(t)P_{t-1}\varphi(t)}\{y(t) - \varphi^T(t)\hat{\theta}(t-1)\}$$

$$P_t = \frac{1}{\alpha}[P_{t-1} - \frac{P_{t-1}\varphi(t)\varphi^T(t)P_{t-1}}{\alpha + \varphi^T(t)P_{t-1}\varphi(t)}]$$

where $0 < \alpha < 1$.

a). Implement the above algorithm using MATLAB. (MATLAB already has this algorithm in its library, but you'd better code it on your own.) Test out your code with Data 1-a in the 2.160 locker. Plot the estimated parameter values against time *t*. Repeat the computation for different values of forgetting factor α, and discuss the effect of α.

b). Use Data 1-b also in the course locker, and plot the parameter values. Examine the P matrix, and discuss the results.

The question below is a challenging problem, which requires a bit more theoretical understanding. If you can solve it, that will be great. You will get an extra credit for the work. Do not feel bad, although you cannot complete it.

c). The data set in Part b) of the above question is a special case where $\{\varphi(t), t = 1, 2\cdots\}$ lies in a hyperplane of dimension lower than that of parameter vector θ. Theoretically show that P_t diverges. Discuss the implication of this observation.