

## 2.160 Identification, Estimation, and Learning

3-0-9 H-Level Graduate Credit  
Spring 2006  
General Course Information

Prerequisite	2.151 or similar subject
Instructor in charge	Professor H. Harry Asada Ford Professor of Mechanical Engineering <a href="mailto:asada@mit.edu">asada@mit.edu</a> , Room 3-346, x3-6257
Office Hours	Monday and Wednesday, 2:30 pm ~ 3:00 pm, and Tuesday 4:00 pm ~ 5:00 pm
Course Secretary	Amy Shea, <a href="mailto:amyshea@mit.edu">amyshea@mit.edu</a> , Room 3-348, x3-2204
Class	Monday and Wednesday, 1:00 pm ~ 2:30 pm, Room 1-273
Lecture Notes	Lecture notes will be provided for every lecture except for research survey lectures. See the table of contents below.

### References

Most of the course materials have been developed based on the following references.

1. "System Identification: Theory for the User, Second Edition", Lennart Ljung, Prentice-Hall 1999, ISBN 0-13-656695-2
2. "Adaptive Filtering, Prediction, and Control", Graham Goodwin and Kwai Sang Sin, Prentice-Hall 1984, ISBN 0-13-004069-x, QA402.G658
3. "Model Selection and Multimodel Inference: A Practical Information-Theoretic Approach, Second Edition", Kenneth Burnham and David Anderson, Springer 1998, ISBN 0-387-95364-7, QH323.5 B87
4. "Introduction to Random Signals and Applied Kalman Filtering, Third Edition", Robert Brown and Patrick Hwang, Wiley 1997, ISBN 0-471-12839-2, TK5102.9.B75

These books are available in Room 3-348 for your reference. You can borrow these books for a limited period of time. No textbook will be used for this subject.

### Grading

First exam*, (12:30 pm – 2:30 pm, April 3, 2006)	30%
Second exam*, (12:30 pm – 2:30 pm, May 17, 2006)	30%
Homework Assignment (8 ~ 9 assignments)	20 %
Term project (Suggested topics and guidelines will be provided.)	20%
Total	100%

\* Both exams are 120 minutes, 30 minutes longer.

## 2.160 Identification, Estimation, and Learning Spring 2005 Lecture Schedule

1. 2/8-W Introduction
2. 2/13-M Recursive Least Square algorithms
3. 2/15-W Properties of RLS
4. 2/21-T Random processes, Active noise cancellation (Rescheduled for Friday 2/17)
5. 2/22-W Discrete Kalman Filter-1 (Rescheduled for Friday 2/24)
6. 2/27-M Discrete Kalman Filter-2
7. 3/1-W Continuous and Extended Kalman Filters
8. 3/6-M Prediction modeling of linear systems
9. 3/8-W Model structure of linear time-invariant systems
10. 3/13-M Time series data compression, Laguerre series expansion
11. 3/15-W Non-linear models, Function approximation theory, Radial basis functions
12. 3/20-M Neural networks
13. 3/22-W Error back propagation algorithm
- 3/27&29 Spring Break
14. 4/3-M Mid-Term Exam
15. 4/5-W Time-frequency analysis
16. 4/10-M Wavelet transforms
17. 4/12-W Multi-resolution analysis, Daubechies' wavelets
- 4/17-M Patriots Day, No class
18. 4/19-W Perspective of system identification, Informative data sets and consistency
19. 4/24-M Persistent excitation, frequency domain analysis
20. 4/26-W Asymptotic distribution of estimates, central limit theorems
21. 5/1-M Experiment design, Pseudo random binary signals
22. 5/3-W Maximum Likelihood Estimate
23. 5/8-M Cramer-Rao Lower Bound and best unbiased estimate
24. 5/10-W Kullback-Leibler information distance, Akaike's Information Criterion
25. 5/15-M Model structure selection and system order estimate
26. 5/17-W End-of-Term Exam

**Table of Contents**  
2.160 Lecture Notes  
Identification, Estimation, and Learning

1. Introduction

Physical modeling vs. Black-box modeling  
System Identification in a Nutshell  
Applications

**Part 1 ESTIMATION**

2. Parameter Estimation for Deterministic Systems

2.1 Least Squares Estimation  
2.2 The Recursive Least-Squares Algorithm  
2.3 Physical meanings and properties of matrix  $P$   
    *Geometric interpretation of matrix  $P^{-1}$ .*  
2.4 Initial Conditions and Properties of RLS  
2.5 Estimation of Time-varying Parameters  
2.6 Orthogonal Projection  
2.7 Multi-Output, Weighted Least Squares Estimation

3. Introduction to Random Variables and Random Processes

3.1 Random Variables: A Review  
3.2 Random Process  
    Characterization of a random process  
3.3 Application: Adaptive Noise Cancellation

4. Kalman Filtering

4.1 State Estimation Using Observers  
4.2 Multivariate Random Processes  
4.3 State-Space Modeling of Random Processes  
4.4 Framework of the Kalman Filter  
    Optimal State Estimation Problem  
4.5 The Discrete Kalman Filter as a Linear Optimal Filter  
    4.5.1 The Kalman Gain  
    4.5.2 Updating the Error Covariance  
    4.5.3 The Recursive Calculation Procedure for the Discrete Kalman Filter  
4.6 Anatomy of the Discrete Kalman Filter  
4.7. Continuous Kalman Filter  
    4.7.1 Converting the Discrete Filter to a Continuous Filter  
    4.7.2 The Matrix Riccati Equation  
4.8 Extended Kalman Filter  
    4.8.1 Linearized Kalman Filter  
    4.8.2 The Extended Kalman Filter.  
4.9 Convergence Analysis  
    4.9.1 Steady-State Solution  
    4.9.2 Fraction Decomposition  
    4.9.3 Convergence Properties of Scalar

## **Part 2 REPRESENTATION AND LEARNING**

### **5 Prediction Modeling of Linear Systems**

- 5.1 Impulse Response and Transfer Operator (Review)
- 5.2 Z-Transform (Review)
- 5.3 Noise Dynamics
- 5.4 Prediction

### **6 Model Structure of Linear Time Invariant System**

- 6.1 Model Sets
- 6.2 A Family of Transfer Function Models
  - 6.2.1 ARX Model Structure
  - 6.2.2 Linear Regressions
  - 6.2.3 ARMAX Model Structure
  - 6.2.4 Pseudo-linear Regressions
  - 6.2.5 Output Error Model Structure
- 6.3 State Space Model
- 6.4 Times-Series Data Compression
  - 6.4.1 Continuous-Time Laguerre Series Expansion
  - 6.4.2 Discrete-Time Laguerre Series Expansion

### **7 Nonlinear Models**

- 7.1 Nonlinear Black-Box Models
- 7.2 Function Approximation Theory and Local Basis Functions
- 7.3 Tuning of Local Basis Function Networks
  - Non-adaptive methods
  - Adaptive Methods

### **8 Neural Networks**

- 8.1 Physiological Background
- 8.2 Stochastic Approximation
- 8.3 Multi-Layer Perceptrons
- 8.4 The Error Backpropagation Algorithm
  - 8.4.1 The Main Algorithm
  - 8.4.2 Stabilizing Techniques

### **9 Wavelet Transforms**

- 9.1 Mathematical Background
  - Review of Hilbert Space
  - Parseval's Theorem
- 9.2 Gabor Transform: A Windowed Fourier Transform
- 9.3 Wavelet Transform
  - Wavelet Admissibility Conditions
- 9.4 Inverse Wavelet Transform
- 9.5 Discrete Wavelet Transform and Dyadic Sampling Grids
- 9.6 Multiresolution Analysis
- 9.7 Generating Mother Wavelets
- 9.8 Daubechies' Wavelets

### **Part 3 SYSTEM IDENTIFICATION THEORY**

#### **10 Frequency Domain Analysis**

- 10.1 Frequency Response of Sampled-Data Systems (Review)
- 10.2 Periodogram and Discrete Fourier Transform
- 10.3 Signal Spectra
- 10.4 Applying Spectral Analysis to System Identification

#### **11 Informative Data Sets and Consistency**

- 11.1 Perspective of System Identification Theory
- 11.2 Informative Data Sets
- 11.3 Consistency of Prediction Error Based Estimate
- 11.4 Frequency Domain Analysis of Consistency
- 11.5 Persistence of Excitation
- 11.6 Informative Experiments

#### **12 Asymptotic Distribution of Parameter Estimates**

- 12.1 Overview
- 12.2 Central Limit Theorems.
- 12.3 Estimate Distribution
- 12.4 Expression for the Asymptotic Variance
- 12.5 Frequency-Domain Expressions for the Asymptotic Variance

#### **13 Experiment Design**

- 13.1 Review of System ID Theories for Experiment Design
  - Key Requirements for System ID
- 13.2 Design Space of System ID Experiments
- 13.3 Input Design for Open-Loop Experiments
- 13.4 Practical Requirements for Input Design
- 13.5 Pseudo random binary inputs
- 13.6 Sinusoidal Inputs.

#### **14. Maximum Likelihood**

- 14.1 Principle
- 14.2 Likelihood Function for Probabilistic Models of Dynamic Systems
- 14.3 The Cramer-Rao Lower Bound
- 14.4 Best Unbiased Estimators for Dynamical Systems.

#### **15 Information Theory of System Identification**

- 15.1 Overview
- 15.2 The Kullback Leibler Information Distance
- 15.3 Re-formulating the Kullback-Leibler Distance
- 15.4 Computation of Target T
- 15.5 Akaike's Information Criterion (AIC)