This problem focuses on the design of a MIMO controller via classical techniques, by attempting to decouple the plant and then control it via a pair of approximately independent compensators. We also introduce nonlinearities such as friction and quantization which make real-world designs more challenging than most textbook problems. Thus this design exercise will allow you to develop a feel for the application of digital control in a realistic setting.

This problem studies the design of the motion control system for a wafer stepper. This problem is based upon the stage of the GCA machine which I brought into class. Some of the control issues of this stage are highlighted in the handout of Ch. 15 from *Principles of Feedback Control, Vol. 2*, G. Biernson, John Wiley and Sons, 1988.

The problem is broken into several parts. In part 1, we study a linear model and ask you to develop controllers which will move the stage and settle it to the nanometer level in a “reasonable” amount of time. In part 2, we include floor vibration and the nonlinearity of sensor quantization, and study their effect on positioning errors. Finally, part 3 includes the additional nonlinearities of bearing friction and actuator saturation. Here you need to develop control strategies to deal with these nonlinearities. You also need to consider how to generate the reference trajectories as well as any feedforward signals for the stage commands. This problem is open-ended in that there is no one correct solution; try to think creatively about the best ways to solve the problem.

The system which we will study is shown in Figure 1. This model is one-dimensional in the sense that we model all of the masses as able to move in only the $x$-direction. Springs and dampers connected between elements are sensitive to the relative $x$-motion of their ends, and forces are applied in the $x$-direction.

The wafer is carried on the mirror $M_4$, which is mounted to the fine stage $M_3$ via the spring $k_4$ and damper $b_4$. These elements model the relatively stiff kinematic coupling which connects the mirror to the fine stage. The fine stage moves on relatively soft flexures $k_3$ and $b_3$ with respect to the coarse stage $M_2$ as driven by the input force $F_2$. In the real system this force is supplied by a voice coil actuator, which is not modeled in any further detail herein. The coarse stage rides on plain bearings; in Parts 1 and 2 we will model these as frictionless; in Part 3 we will include the significant Coulomb friction present in these bearings. The coarse stage is driven by force $F_1$ with respect to the machine base $M_1$. In the real system, this force is supplied by a rotary motor through a rack-and-pinion drive, but this mechanism is not modeled here. The machine base is supported on spring $k_1$ and damper $b_1$. These model the vibration isolation legs which are present on the real stepper. In order to isolate well, these elements are made to be compliant. However, this introduces the problem that the base is driven by the stage input forces, as you will see in more detail as you work through your solution.
The two system outputs are $y_1$, which is the distance between the coarse and fine stages, and $y_2$, which is the distance between the machine base and the mirror. In practice, $y_1$ is measured with a linear differential variable transformer (LVDT) and $y_2$ is measured with a laser interferometer. Since the system lens is modeled as rigidly attached to the machine base, and the wafer is carried on the mirror, the output $y_2$ is the critical output for control purposes. The details of the lithography process require that the errors in $y_2$ from its setpoint are less than 10 nanometers RMS in a 500 msec exposure interval after each step. Since it is carried on flexures, the travel of the fine stage is limited; in all trajectories, the output $y_1$ must be strictly less than $\pm 200 \mu m$ (we view the fine stage as having hard stops outside this range, but do not model these dynamically).

The main task of the stage system is to take repeated 20 mm steps as frequently as possible while maintaining an exposure window of 500 msec during which time the RMS position error spec is maintained. In each of the sections below, we state the model assumptions. Under these constraints, design a controller which meets the specifications. Tell us how you designed the controller, what gives you confidence that it will meet the specs, and include relevant simulation outputs that show the specs are met. Also tell us what is the maximum step rate at which you can meet the specifications.

One further note on numerics: This system is 8th order, and with the controllers the order will be perhaps double that. There are also widely spaced singularities (i.e., it’s “stiff”). So you will have to be very careful with the numerics of any of your simulations. Do reasonableness tests on any of your data to be sure you’re not seeing a numerical artifact.
Part 1: In this section, the system model is linear; the plain bearings are assumed to be frictionless, and the actuators can apply any requested force. The only nonlinear constraint is that the fine stage relative excursion $y_1$ must be maintained at less than $±200 \text{ µm}$ at all times. We assume no base vibration, i.e., $\ddot{x}_0 = 0$. The linear model is given by the system matrices $A_1$, $B_1$, $C_1$, and $D_1$. These matrices and associated parameters for this and the other parts of this problem are given in the file part1.m. (This and the other necessary files are available through the course web page.)

The task here is to take repeated $±20$ mm steps (these steps can then be generated by a square wave signal source). Between each step, there must be a 500 msec exposure window during which time the error in $y_2$ is held to less than 10 nm RMS. This error must be met in steady-state when the startup transients have decayed; it need not be met during startup. Please let us know whether your design meets spec during startup.

a) Design a linear controller to meet the specs given above. The controller has access to the measurements $y_1$ and $y_2$ but cannot use any other system outputs. The controller can drive the two input forces $F_1$ and $F_2$. In your design you must also plan for reasonable robustness. That is, while you know the model perfectly here, do not unrealistically take advantage of this in your design. For example, using notch filters to exactly cancel the plant modes is not reasonable. You should plan that the resonances may change slightly in frequency and damping over time. As part of the controller design, we expect you to generate appropriately-shaped reference trajectories (this is likely to be necessary to meet spec in any case). You may also generate feedforward signals to drive the stage command forces in order to more rapidly converge on the final value. Any such feedforwards have to work in the presence of plant uncertainties, nonlinearities, etc. You need to demonstrate that your design works to spec even when plant parameters vary randomly by $±10\%$.

In solving this design problem, we ask that you take a classical approach. That is, we want you to attempt to implement transformations on the plant inputs and outputs which allow you to decouple the two inputs and outputs and then drive these with two independent compensators. You are free to also develop your own approach to this problem as long as it lies within the classical design tools of using series compensation of an approximately decoupled plant. Please tell us how and why you picked your approach. Tell us how you designed your controller: What is the design philosophy? What calculations were required? Why do you expect it to meet specs? What design iterations did you go through? How robust is your controller design to variations in the plant resonance locations, gain changes, and phase shifts? You also need to show us Bode or Nyquist plots that indicate the design is stable and has good phase margin.

b) Simulate your controller driving the stage system. Provide plots of the system response which convince us that your design meets the required specs. What is the maximum step rate (exposures per minute) that your design supports? What limits you from going faster than this? Why? Do some additional simulations where plant parameters are varied by $±10\%$ while the controller design is held fixed in order to demonstrate robustness. Note that since this section is linear, the plant can be readily simulated using the linear simulation tools in Matlab, and that you don’t necessarily need to use a Simulink model which would run slower.

Part 2: In this section, we make the design problem somewhat more realistic by adding in floor vibration and sensor quantization. The underlying system model remains linear, except for
the constraint on fine stage motion. The floor vibration model is a set of two sinusoids, i.e.,
\[ \ddot{x}_0(t) = e_1 \cos \omega_1 t + e_2 \cos \omega_2 t \] (1)

The variables \( e_1, e_2, \omega_1, \) and \( \omega_2 \) are given in the file part1.m (where \( w \) is used in place of \( \omega \)).

The sensor quantization is as follows. The LVDT output \( y_1 \) is quantized at the 25 nm level (this represents an approximately 14-bit A/D converter on this channel). The interferometer output \( y_2 \) is quantized at the 2.5 nm level (this is a typical resolution for a laser interferometer system).

A Simulink representation of this model incorporating the floor motion and sensor quantization is given in the file part2.mdl.

a) For this system, design a controller which meets the specs. How did you address the base vibrations? How did you address the sensor quantization? Include the outputs of simulations which show that the specs have been met.

**Part 3:** In this section, we add in the constraints that the actuators have a maximum force capability and that the plain bearings have a significant friction term. Specifically, assume that \( F_1 \) is limited to \( \pm F_{1max} \) and that \( F_2 \) is limited to \( \pm F_{2max} \). For this problem, assume that \( F_{1max} = 200 \) N and \( F_{2max} = 50 \) N. Also, assume that the coarse stage plain bearings have a Coulomb friction of \( \pm 10 \) N. That is, the controller force (now termed \( F_{1c} \)) has subtracted from it a term equal to \( 10 \) N times the sign of the relative velocity \( q_4 \), i.e.,

\[ F_1 = F_{1c} - 10 \text{ signum}(q_4) \] (2)

A Simulink representation of this model incorporating the actuator limits and bearing friction is given in the file part3.mdl. Note that since we need an output of \( q_4 \) to drive the friction term, the plant in Part 3 uses augmented output matrices \( C_2 \) and \( D_2 \) as defined in part1.m.

One caution: the friction model introduces numerical chatter in the force applied to the stage. You will need to run the simulation at a sufficiently fine update time in order to reduce the effect of this chatter, so that artifacts are not significant in the system outputs. The simulation of Coulomb friction if problematic. If you can find a better way to include this term in your simulations, please change the block diagram to do so, and include information to this effect in your lab report.

a) Under these additional nonidealities, and including all of the nonideal terms from the previous two parts, design a controller to meet the system specifications. How did you address the bearing friction term? How did you address the actuator saturation? Include the outputs of simulations which show that the specs have been met.

**Plant linear model** The linear dynamics for the system shown in Figure 1 may be expressed as

\[ \dot{q} = A_1 q + B_1 u \]
\[ y = C_1 q + D_1 u \]

where the eight state variables are defined as

\[ q_1 = x_1 - x_0 \]
\[ q_2 = \dot{q}_1 \]
\[ q_3 = x_2 - x_1 \]
\[ q_4 = \dot{q}_3 \]
\[ q_5 = x_3 - x_2 \]
\[ q_6 = \dot{q}_5 \]
\[ q_7 = x_4 - x_3 \]
\[ q_8 = \dot{q}_7 \]

The state variables are defined in terms of relative position and velocity in order that the floor acceleration \( \ddot{x}_0 \) enters conveniently. This formulation is also convenient when the nonlinear friction term is added subsequently. In terms of these state variables, the system matrices and numerical parameters are as given in the MATLAB `part1.m` file.

**Definition of RMS error** For a continuous-time signal \( e(t) \), the RMS value is defined as:

\[ e_{RMS} = \sqrt{\frac{1}{T} \int_0^T e^2(t) dt} \quad (3) \]

You will need to numerically compute the RMS error from your simulation data. Tell us how you carry out this approximation. As a point of reference, a Gaussian random waveform will have a peak-to-peak value of about 6 times its RMS value, whereas a pure sinusoid has a peak-to-peak value of \( 2\sqrt{2} \) times its RMS value.