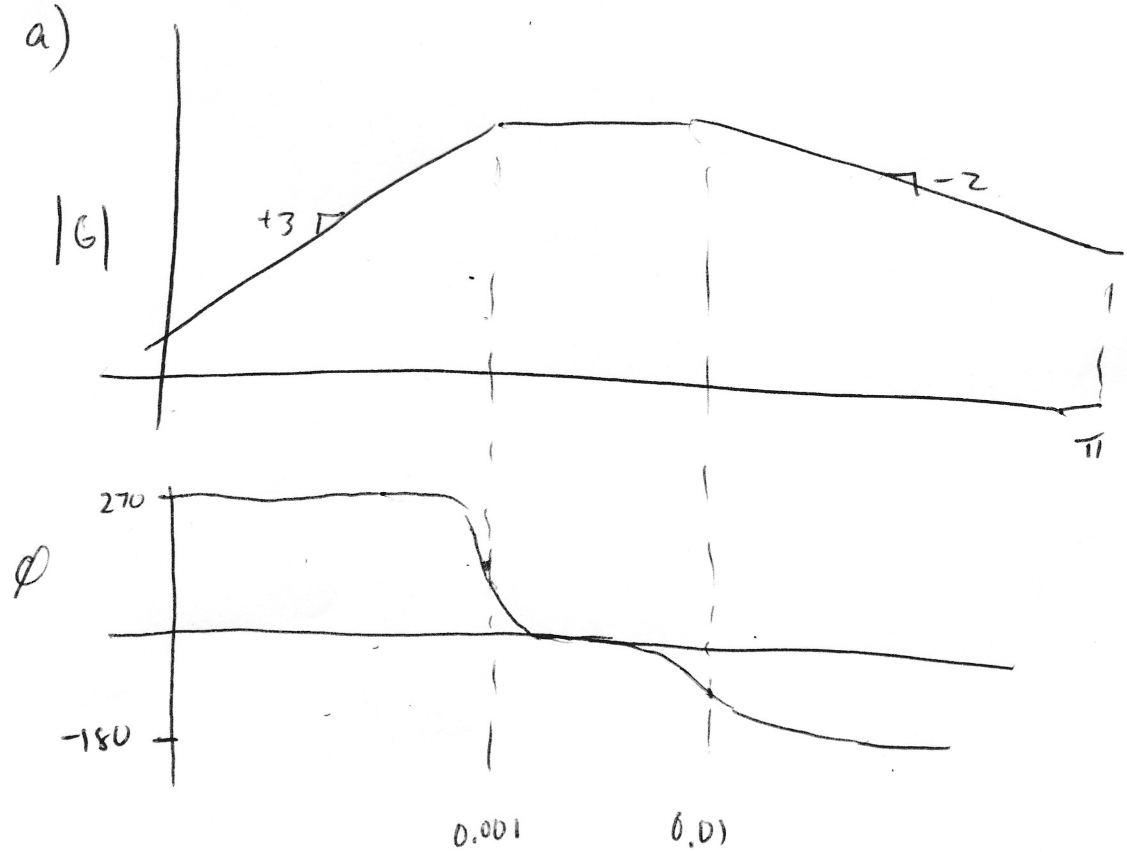
2.171 Problem Set #6 Solutions

Problem 1

a)

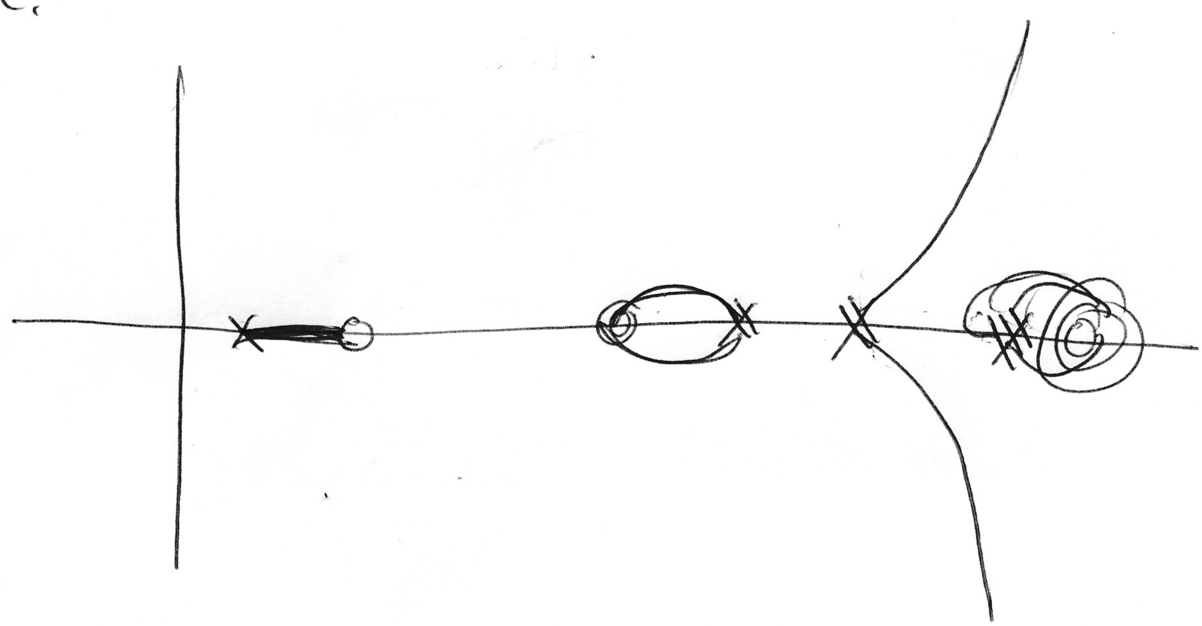


b) The bandwidth limits are set by the high frequency gain and the sampling frequency.

A lead, double-lag compensator was used. The lead compensator provides some extra high frequency phase margin, while the two lower frequency lag compensators reduce the phase at lower frequencies such that the phase is within acceptable limits.

$$G(z) = \underbrace{\frac{(z-1)^3}{(z-.999)^3(z-.99)^2}}_{\text{Plant}} \underbrace{\frac{z-.2}{z-.05}}_{\text{lead}} \underbrace{\frac{(z-.9)^2}{(z-.94)^2}}_{\text{double lag}} \cdot \underbrace{1.0501}_{\text{gain}}$$

c.



[illegible]

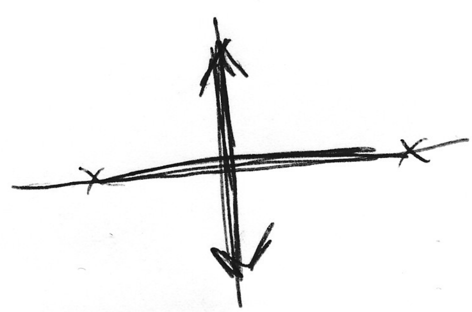
MATLAB confirms the plot

Problem 2

a. $m\ddot{x} - k_1 x = k_2 i$

$m s^2 X(s) - k_1 X(s) = k_2 I(s)$

$$\frac{X(s)}{I(s)} = \frac{k_2}{m s^2 - k_1} = \frac{0.4}{0.02 s^2 - 20}$$



b. $G(z) = \frac{z-1}{z} Z\left\{\frac{G(s)}{s}\right\}$

$$\frac{G(s)}{s} = 0.4 \frac{1}{(0.02 s^2 - 20)} \frac{1}{s} = \frac{A}{s - 31.6} + \frac{B}{s + 31.6} + \frac{C}{s}$$

$A = 0.01,\ B = 0.01,\ C = -0.02$

Using the lookup tables in Appendix B of FPW

$$G(z) = \frac{0.004135 z + 0.004135 z}{z^2 - 2.414 z + 1}$$

c. $t_r \leq 0.1\,s$, $t_s \leq 0.4\,s$, overshoot $\leq 20\%$

$$\omega_n \geq \frac{1.8}{t_r} \Rightarrow \omega_n \geq 18 \text{ rad/sec}$$

$$\zeta\omega_n \geq \frac{4.6}{t_s} \Rightarrow \zeta \geq .638$$

$$\text{overshoot} \leq e^{(-\pi\zeta/\sqrt{1-\zeta^2})}$$

$$0.2 \leq e^{-\pi\zeta/\sqrt{1-\zeta^2}}, \quad \zeta \geq .46$$

The specifications were met using SISOTOOL

~~[illegible]~~

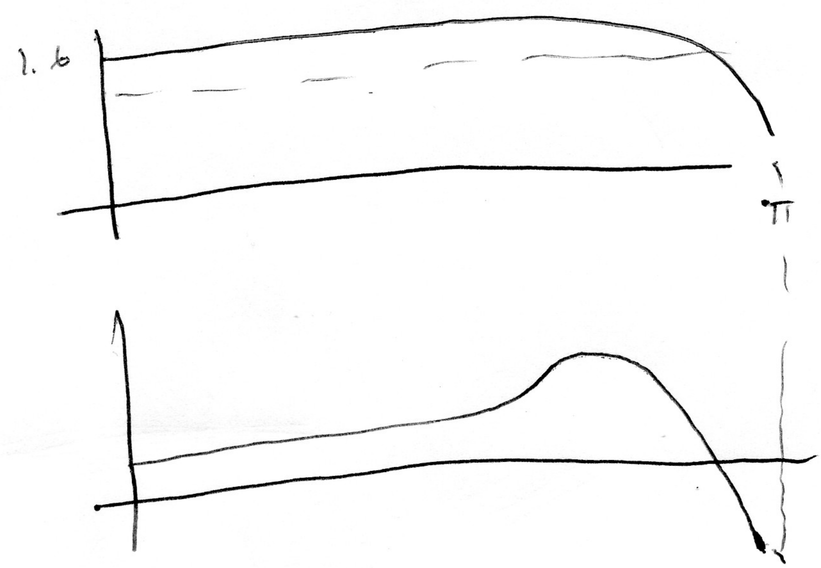
$$G_c(z) = 5600\,\frac{z - .9983}{z - .986}$$

The necessary compensator is a lead compensator

Problem 3

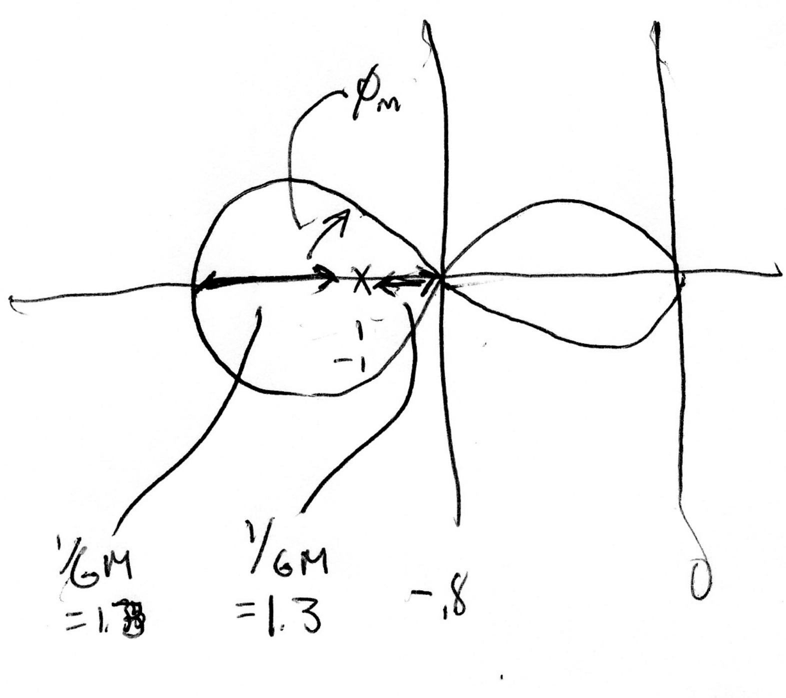
(a) see attached, the design specs are evident in the error response

(b)



The phase margin is 26°

The gain margins can be represented on the Nyquist plot

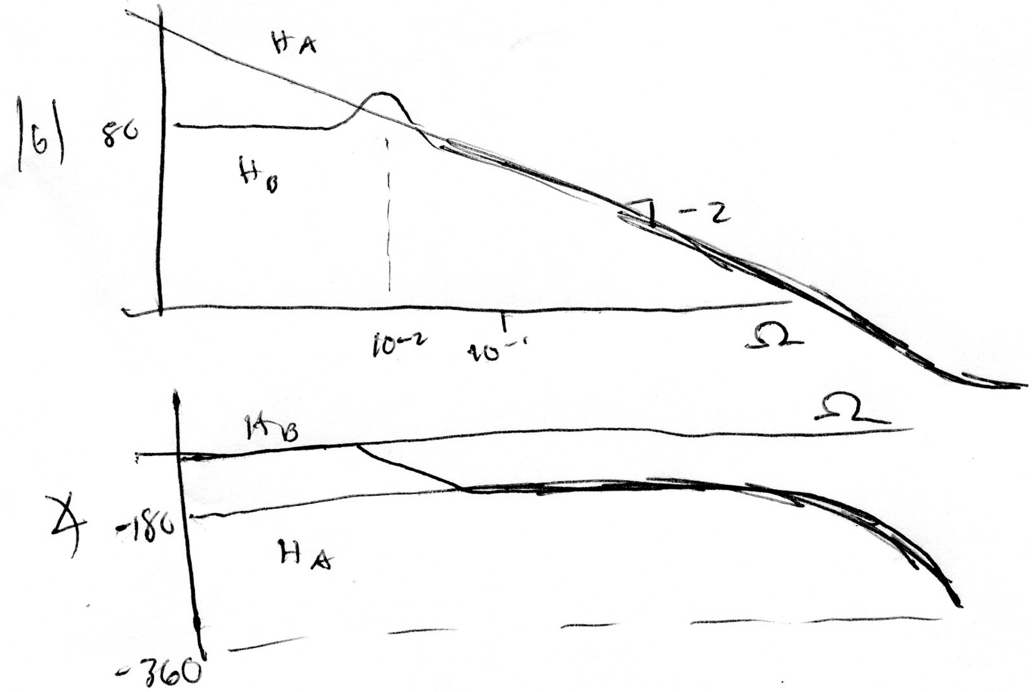


(c)

$$G(z) = 4000 \frac{(z-.99)}{z-.94}$$

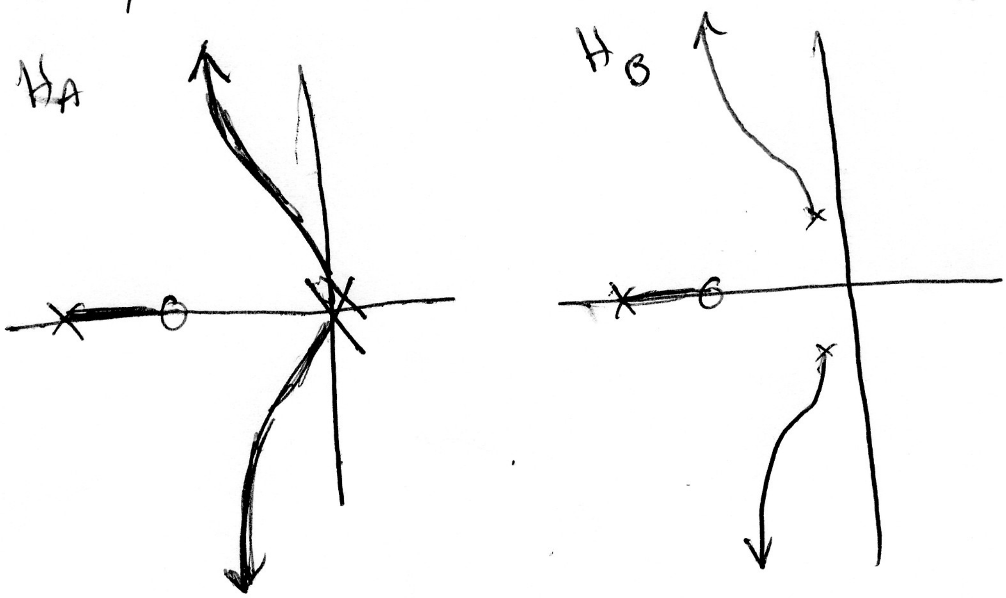
Problem 4

(a)



As seen graphically, H_A and H_B are nearly identical after $\Omega > 0.01$ rad/sec

(b)



(c) $G(z) = 0.02 \dfrac{z-0.95}{z-0.8}$

$$\alpha = \frac{1-0.8}{1-0.95} = 4$$

$$\phi_{gain} = \sin^{-1}\left(\frac{\alpha-1}{\alpha+1}\right) = 36.9°$$

Problem 5

(a)

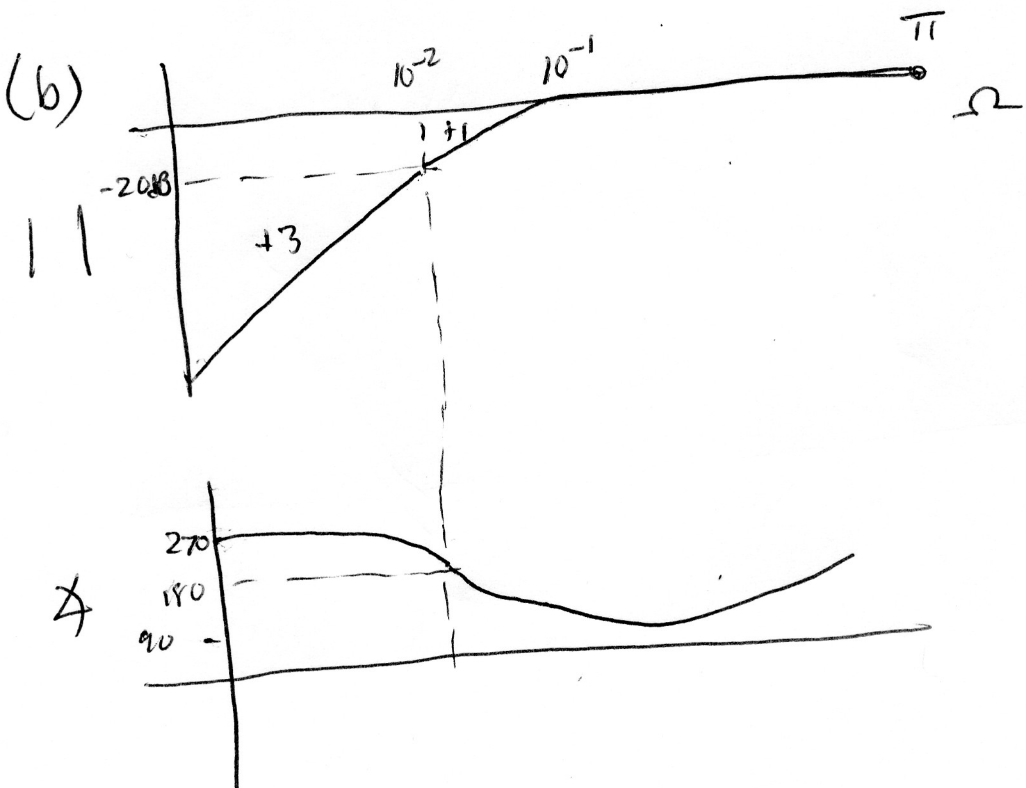
$$H_1(z) = \frac{K\frac{z-1}{z-a}}{1+K\frac{z-1}{z-a}\frac{(z-1)^2}{(z-b)^2}} = \frac{K(z-1)(z-b)^2}{(z-a)(z-b)^2+K(z-1)^3}$$

Zeros at 1, b, b

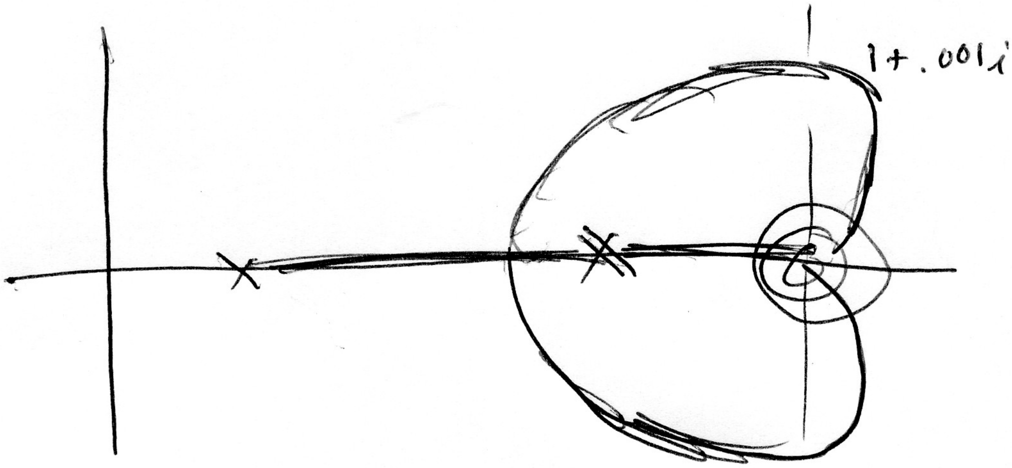
$$H_2(z) = \frac{K}{1+K\frac{z-1}{z-a}\frac{(z-1)^2}{(z-b)^2}} = \frac{(z-a)(z-b)^2}{(z-a)(z-b)^2+K(z-1)^3}$$

Zeros at a, b, b

(b)

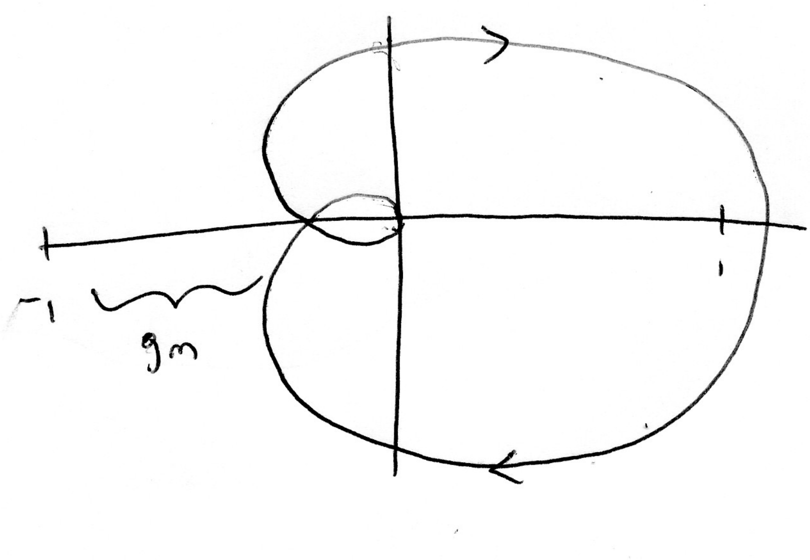


c)



d) At 10^{-1} rad/sample on the Bode plot, the phase is 180° and the gain is -20dB, thus $K \leq 10$

e)



(f)

Lag compensation can be used to meet the phase margin design spec.

Using sisotool,

$$G_c(z) = 30\,\frac{(z-.8)(z-.95)}{(z-.12)(z-.99)}$$