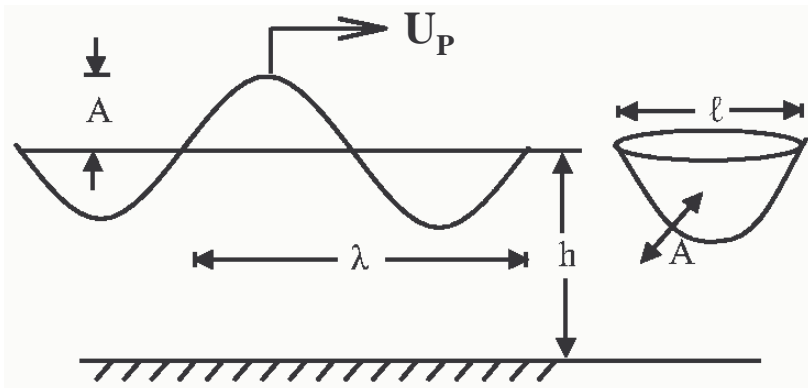


2.20 - Marine Hydrodynamics
Lecture 21

6.9 Wave Forces on a Body



$$U = \omega A$$

$$Re = \frac{U\ell}{\nu} = \frac{\omega A\ell}{\nu}$$

$$K_c = \frac{UT}{\ell} = \frac{A\omega T}{\ell} = 2\pi \frac{A}{\ell}$$

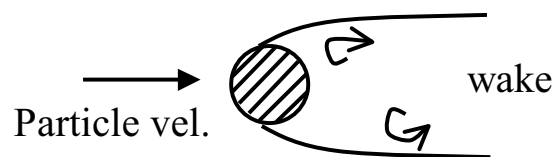
$$C_F = \frac{F}{\rho g A \ell^2} = f\left(\underbrace{\frac{A}{\lambda}}_{\text{Wave steepness}}, \underbrace{\frac{\ell}{\lambda}}_{\text{Diffraction parameter}}, Re, \frac{h}{\lambda}, \text{roughness}, \dots\right)$$

6.9.1 Types of Forces

1. **Viscous forces** Form drag, viscous drag = $f(R_e, K_c, \text{roughness}, \dots)$.

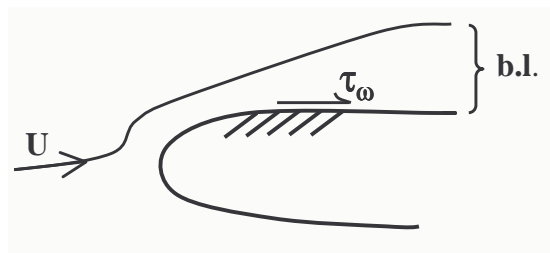
- *Form drag* (C_D)

Associated primarily with flow separation - normal stresses.



- *Friction drag* (C_F)

Associated with skin friction τ_w , i.e., $\vec{F} \sim \iint_{\substack{\text{body} \\ \text{(wetted surface)}}} \tau_w dS$.



2. Inertial forces Froude-Krylov forces, diffraction forces, radiation forces.

Forces arising from potential flow wave theory,

$$\vec{F} = \iint_{\text{body (wetted surface)}} p \hat{n} dS, \text{ where } p = -\rho \left(\frac{\partial \phi}{\partial t} + gy + \underbrace{\frac{1}{2} |\nabla \phi|^2}_{=0, \text{ for linear theory, small amplitude waves}} \right)$$

For linear theory, the velocity potential ϕ and the pressure p can be decomposed to

$$\begin{aligned} \phi &= \underbrace{\phi_I}_{\text{Incident wave potential (a)}} + \underbrace{\phi_D}_{\text{Diffracted wave potential (b.1)}} + \underbrace{\phi_R}_{\text{Radiated wave potential (b.2)}} \\ -\frac{p}{\rho} &= \frac{\partial \phi_I}{\partial t} + \frac{\partial \phi_D}{\partial t} + \frac{\partial \phi_R}{\partial t} + gy \end{aligned}$$

(a) Incident wave potential

- *Froude-Krylov Force approximation* When $\ell \ll \lambda$, the incident wave field is not significantly modified by the presence of the body, therefore ignore ϕ_D and ϕ_R . Froude-Krylov approximation:

$$\left. \begin{aligned} \phi &\approx \phi_I \\ p &\approx -\rho \left(\frac{\partial \phi_I}{\partial t} + gy \right) \end{aligned} \right\} \Rightarrow \vec{F}_{FK} = \iint_{\text{body surface}} \underbrace{-\rho \left(\frac{\partial \phi_I}{\partial t} + gy \right)}_{\equiv p_I} \hat{n} dS \leftarrow \begin{array}{l} \text{can calculate knowing (incident)} \\ \text{wave kinematics (and body geometry)} \end{array}$$

- *Mathematical approximation* After applying the divergence theorem, the \vec{F}_{FK} can be rewritten as $\vec{F}_{FK} = - \iint_{\text{body surface}} p_I \hat{n} dS = - \iiint_{\text{body volume}} \nabla p_I dV$.

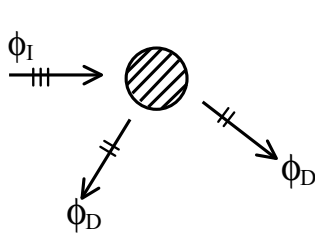
If the body dimensions are very small comparable to the wave length, we can *assume* that ∇p_I is *approximately* constant through the body volume \forall and ‘pull’ the ∇p_I out of the integral. Thus, the \vec{F}_{FK} can be approximated as

$$\vec{F}_{FK} \cong \left(-\nabla p_I \right) \Big|_{\text{at body center}} \iiint_{\text{body volume}} dV = \underbrace{\forall}_{\text{body volume}} \left(-\nabla p_I \right) \Big|_{\text{at body center}}$$

The last relation is particularly useful for small bodies of non-trivial geometry - for 2.20, that is all bodies that do not have a rectangular cross section.

(b) **Diffraction and Radiation Forces**

(b.1) **Diffraction or scattering force** When $\ell \ll \lambda$, the wave field near the body will be affected even if the body is stationary, so that no-flux B.C. is satisfied.

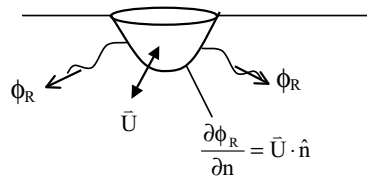


$$\frac{\partial \phi}{\partial n} = 0 = \frac{\partial}{\partial n} (\phi_I + \phi_D)$$

$$\text{or } \frac{\partial \phi_D}{\partial n} = -\frac{\partial \phi_I}{\partial n} \leftarrow \text{given}$$

$$\vec{F}_D = \iint_{\text{body surface}} -\rho \left(\frac{\partial \phi_D}{\partial t} \right) \hat{n} dS$$

(b.2) **Radiation Force - added mass and damping coefficient** Even in the absence of an incident wave, a body in motion creates waves and hence inertial wave forces.



$$\frac{\partial \phi_R}{\partial n} = \vec{U} \cdot \hat{n}$$

$$\vec{F}_R = \iint_{\text{body surface}} -\rho \left(\frac{\partial \phi_R}{\partial t} \right) \hat{n} dS = - \underbrace{m_{ij}}_{\text{added mass}} \dot{U}_j - \underbrace{d_{ij}}_{\text{wave radiation damping}} U_j$$

6.9.2 Important parameters

$$\left. \begin{array}{l} (1) K_c = \frac{UT}{\ell} = 2\pi \frac{A}{\ell} \\ (2) \text{diffraction parameter } \frac{\ell}{\lambda} \end{array} \right\} \begin{array}{l} \text{Interrelated through maximum wave steepness} \\ \frac{A}{\lambda} \leq 0.07 \\ \left(\frac{A}{\ell}\right) \left(\frac{\ell}{\lambda}\right) \leq 0.07 \end{array}$$

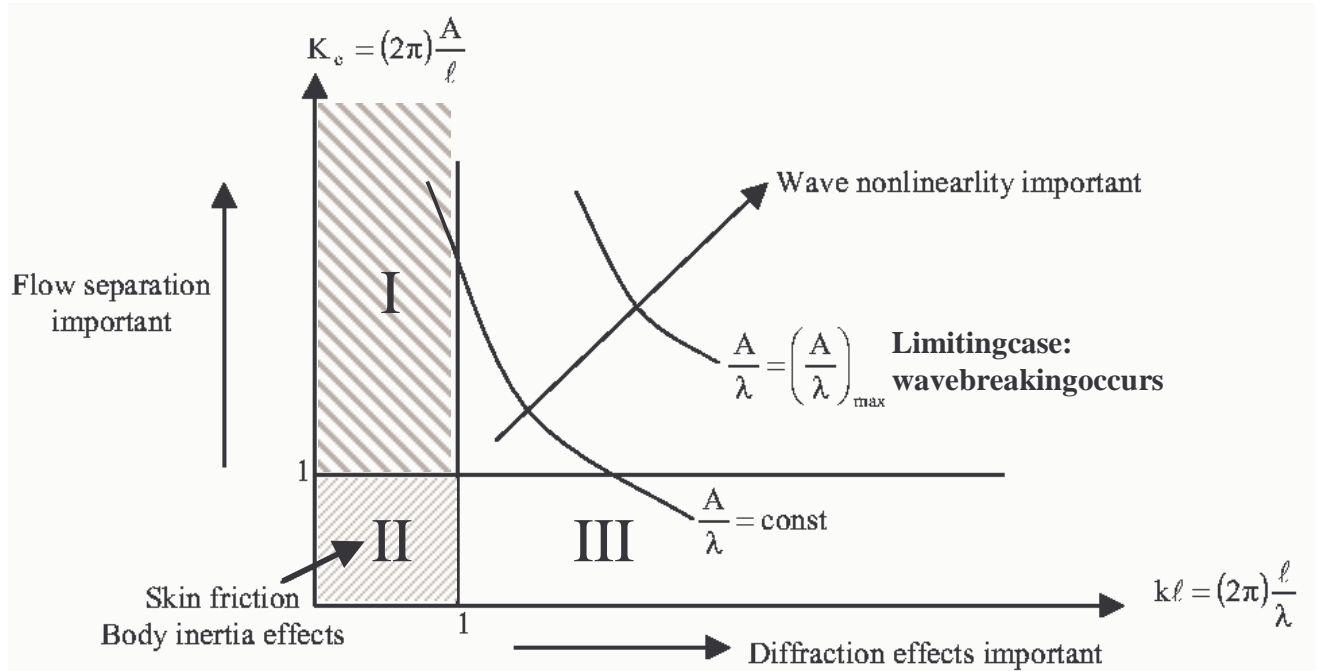
- If $K_c \leq 1$: no appreciable flow separation, viscous effect confined to boundary layer (hence small), solve problem via potential theory. In addition, depending on the value of the ratio $\frac{\ell}{\lambda}$,
 - If $\frac{\ell}{\lambda} \ll 1$, ignore diffraction, wave effects in radiation problem (i.e., $d_{ij} \approx 0$, $m_{ij} \approx m_{ij}$ infinite fluid added mass). F-K approximation might be used, calculate \vec{F}_{FK} .
 - If $\frac{\ell}{\lambda} \gg 1/5$, must consider wave diffraction, radiation ($\frac{A}{\ell} \leq \frac{0.07}{\ell/\lambda} \leq 0.035$).
- If $K_c \gg 1$: separation important, viscous forces can not be neglected. Further on if $\frac{\ell}{\lambda} \leq \frac{0.07}{A/\ell}$ so $\frac{\ell}{\lambda} \ll 1$ ignore diffraction, i.e., the Froude-Krylov approximation is valid.

$$F = \frac{1}{2} \rho \ell^2 \underbrace{U(t)}_{\text{relative velocity}} |U(t)| C_D(R_e)$$

- Intermediate K_c - both viscous and inertial effects important, use Morrison's formula.

$$F = \frac{1}{2} \rho \ell^2 U(t) |U(t)| C_D(R_e) + \rho \ell^3 \dot{U} C_m(R_e, K_c)$$

- Summary



- I. Use: C_D and $F - K$ approximation.
- II. Use: C_F and $F - K$ approximation.
- III. C_D is not important and $F - K$ approximation is not valid.