DEPARTMENT OF MECHANICAL ENGINEERING

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

2.20 Marine Hydrodynamics

Fall 2006 Quiz I

INSTRUCTIONS:

For the problems in section A, fill in the required answers where indicated by ______________, (or in the provided space). When a list of options, [. . .] [. . .] [. . .], is provided select (by circling) all (none, one or more of) the options which apply. For problems in section B, write your solutions in the exam book provided.

Unless otherwise indicated, use gravitational acceleration $\mathrm{g}=10\,\mathrm{m/s^2}$, water density $\rho=10^3\,\mathrm{kg/m^3}$, and kinematic viscosity $\nu=10^{-6}\,\mathrm{m^2/s}$; and air density $\rho=1\,\mathrm{kg/m^3}$ and kinematic viscosity $\nu=10^{-5}\,\mathrm{m^2/s}$. Give all your numerical results in SI ($\mathrm{kg, m, s}$) units. For numerical answers, you MUST give also the proper units (e.g., $5\,\mathrm{m^2/s}$ not 5).

There are a large number of problems and each individual answer can only be worth that many points. DO NOT spend a disproportionate amount of time on any one problem.

YOUR NAME:________________________

SECTION A [50%]

1. A fluid flow is incompressible *only* if **[ρ =constant] [$\partial\rho/\partial t$ =0] [$D\rho/Dt$ =0] [$\partial\rho/\partial t$ =0 and $D\rho/Dt$ =0] [either $\partial\rho/\partial t$ =0 or $D\rho/Dt$ =0]**. For a general (compressible) flow, the equation expressing conservation of mass in terms of density ρ and velocity $\vec{v}$ is_____________. For a certain *steady* flow, it is found that the $\vec{v}$ is everywhere perpendicular to the density gradient $\nabla\rho$, conservation of mass can now be simplified to__________________.

2. For a steady two-dimensional flow with velocity $\vec{v}(x,y)$, a streamline is drawn in the flow field with the normal vector at each point on this line given as $\vec{n}(x,y)$. Then $\vec{v}(x,y)$ and $\vec{n}(x,y)$ must satisfy the condition _______________. The boundary of a **[fixed body] [moving body] [free surface]** is *always* a streamline.

3. A small pressure probe moves with velocity $\vec{U}(\vec{x},t)$ under water. The pressure field in the water is $p(\vec{x},t)$. If for a certain $\vec{U}(\vec{x},t)$, the pressure measured by the probe is found to be a constant (in time), $\vec{U}(\vec{x},t)$ and $p(\vec{x},t)$ must satisfy the condition _______________.

4. The relation between viscous stress and velocity gradient $\tau_{ij} = \mu(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i})$ is obtained under the following assumption(s): **[steady] [incompressible] [irrotational] [linear relationship between viscous stress and rate of strain] [inviscid] [isotropic] [conservative force] [none of above]**. In a two-dimensional flow satisfying above assumption(s), the velocity is given by $\vec{v}(x,y) = (x^2 + y^2, -2xy)$, then $\tau_{xx} =$ _____________, $\tau_{xy} =$ __________. A small flat surface of area A located in the flow at *(x, y)* has an unit normal vector given by $\vec{n} = (n_x, n_y)$. The viscous force on this surface in the *x* direction is given by F_x = _______________________.

5. A model test is performed to estimate the drag force on a *L=160m* long ship moving in water with a speed of *U=10m/s*. The relevant Reynolds number= ________________________ and the Froude number = ___________________. For a 1:10 length scale model test (with the same gravity acceleration and water properties), in order to maintain Reynolds similitude, the velocity of the model should be _________________. However, in order to maintain Froude similitude, the velocity of the model should be ______________. To maintain both Reynolds and Froude similitude in the model test, we need to either change the gravity acceleration to a value of _____ ____________ or find a fluid with kinematic viscosity = ________________.

6. In a certain experiment, a small spherical air bubble is formed in water at a depth *H=1.4m*. The surface tension coefficient for the water/air bubble interface is $\Sigma = 0.07 N/m$. If the radius of the bubble is *R=10*$^{-5}$*m*, the pressure inside the air bubble is given by $p_{bubble} = p_{atmosphere} +$ _______ ______ N/m^2.

7. For ideal fluid flow, indicate true/false for the following statements:

   (a) kinematic viscosity ν =0. [true] [false]

   (b) infinite velocity gradient is allowed. [true] [false]

   (c) kinematic boundary condition requires $\vec{v} = \vec{U}_b$. [true] [false]

   (d) viscous stress is zero everywhere. [true] [false]

8. Kelvin theorem says that $\frac{d\Gamma}{dt} = 0$ following any closed material contour under the condition(s) of

____.

   Under these conditions, a thin vortex tube has at time t_1 a cross-section area A_1, length tube L_1, and circulation Γ_1. If the vorticity ω_1 is uniform across the cross-section, the value of ω_1 should be ω_1=__________________. At a later time t_2, the length of the tube becomes $L_2 = L_1/2$. Now, the tube cross-section area is A_2=_________, the circulation is Γ_2=_______________, and the uniform vorticity in the tube is ω_2=__________________. (*Please write all your answers in terms of* A_1, L_1 *and* Γ_1)

9. Consider the steady flow in a water tunnel of circular cross section. The tunnel has a contraction ratio of 10 so that the radius at the upstream station is $R_1=10m$ and the radius in the test section is $R_2=1m$. Assuming ideal uniform one-dimensional (longitudinal velocity U only) fluid flow and uniform pressure in the cross section, if $p_1=10^5 N/m^2$ and $U_1=0.1m/s$ at the upstream station, then at the test section, U_2= ___________m/s and p_2=_____________ N/m^2.

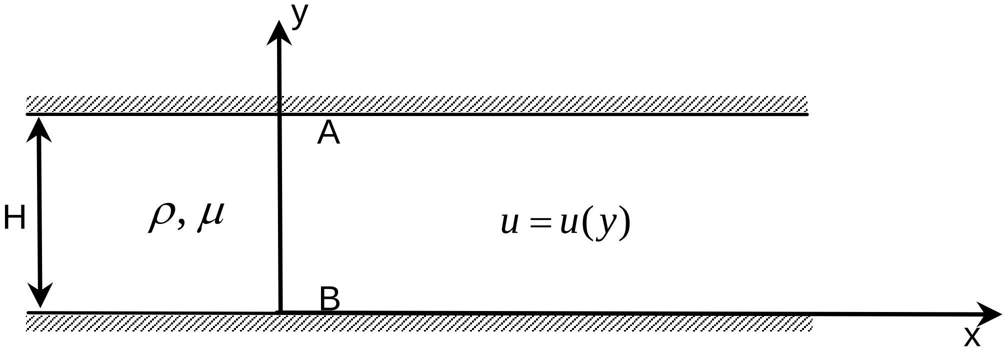
10. The velocity field of a certain two-dimensional flow is $(u,v) = (-4x + 2y,\ Ax + By)$, where A, B are constants. For this flow to be a potential flow, the values of A, B must be A= ____________, and B= ________________________________.

(See Next Page for Section B Problems)

SECTION B [50%]

[Note that in problems with multiple parts, later parts can often be solved without getting earlier parts (completely).]

1. [20%] Consider a two-dimensional, steady, incompressible, viscous flow with velocity $\vec{u} = (u,\ v)$, viscosity μ, density ρ, between two fixed parallel long plates distance *H* apart. The horizontal velocity is known to be a function of *y* only, i.e., *u=u(y)*. Gravity can be ignored.



(a) Write down (i) the continuity equation and (ii) the Navier-Stokes equations in the x- and y-directions describing this flow.

(b) State the kinematic boundary conditions on the upper (*y=H*) and lower plates (*y=0*).

(c) What is the vertical velocity v in the flow?

(d) Using the above, obtain simplified Navier-Stokes equations to show that *dp/dx* is constant.

Use $\frac{dp}{dx} = -C$ for the following:

(e) (i) Solve the simplified Navier-Stokes equation you obtained in (d) using the boundary conditions in (b) to obtain the horizontal velocity profile *u=u(y)*; (ii) find out where (position in y-direction) the horizontal velocity *u* reaches its maximum value; and (iii) express this maximum value in terms of *H*, *C*, and μ.

(f) Find out the shear stress on the upper (τ_t) and lower (τ_b) plates from your solution in (e).

(g) **[*Extra credit*]** Using control volume analysis, confirm your result in (f).

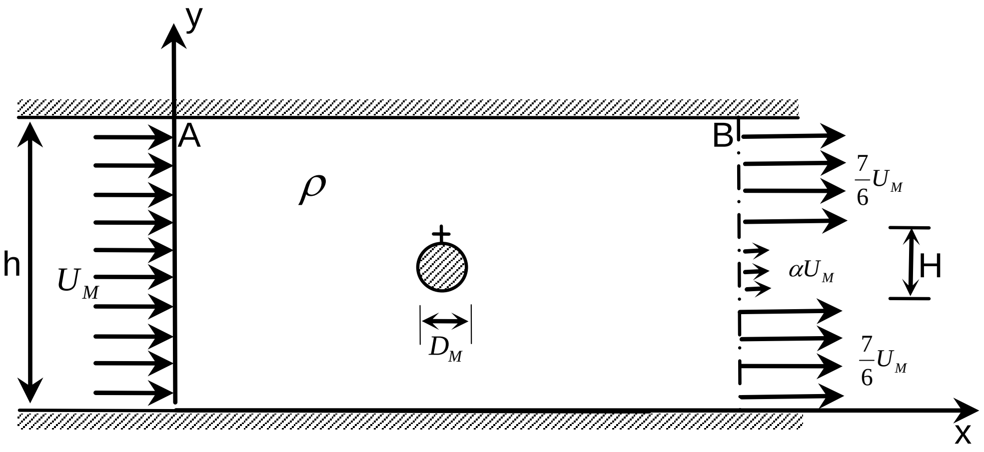
2. [30%] Model tests are performed to find the drag force on a submarine operating in deep water in the presence of a sideward uniform current. The six relevant variables in this problem are the water density ρ, the water viscosity μ, the current speed U, the length of the submarine L, the diameter of the submarine cross-section D, and the drag force F. The full scale values are *L*=L_S=100m, *D*=D_S=10m and *U*=U_S=0.1m/s. The model submarine has a cross-section diameter of D_M=0.2m.

(a) Write down the dimensions of the six relevant variables of the problem;

(b) From Buckingham π theory we know that there are three independent dimensionless variables (π's) for this problem. If $\pi_1 = D/L$, find the other two π's and try to manipulate your answers such that F only appears in one of them while μ only appears in the other.

(c) If ρ and μ are the same in both the full scale and the model, what should be the length of the model L_M and the current speed U_M in the model test in order to obtain geometric and dynamic similitude?

(d) The model tests are performed in a two-dimensional tunnel with a height of *h=2.4m* on the cross section of the submarine as shown below. For a uniform incoming flow with velocity *U*=U_M, the downstream velocity profile can be approximated as shown below, where the height of the wake region is found to be *H=h/6=0.4m*:



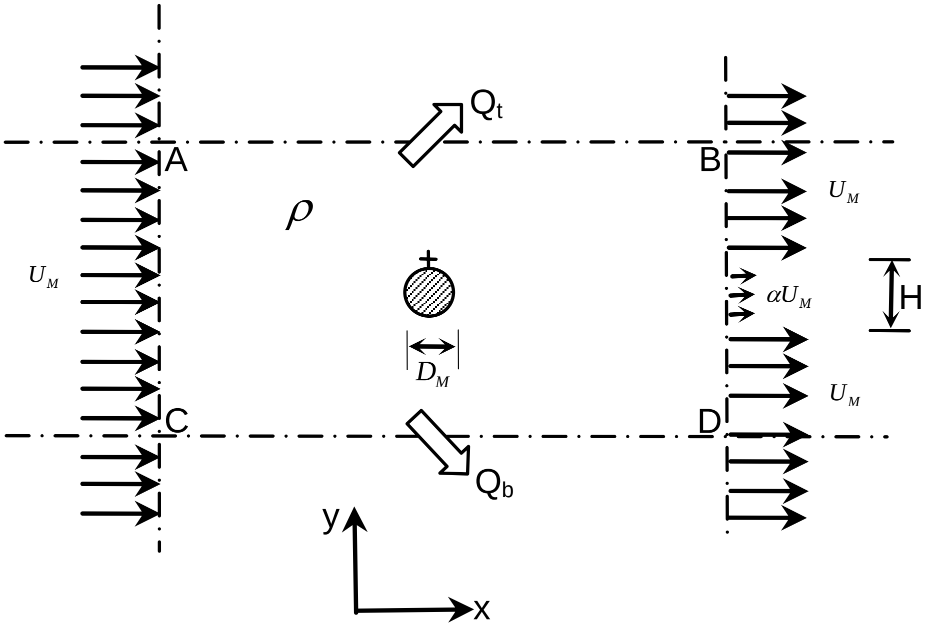
(i) Determine the value of α for the velocity profile in the wake region.

(ii) The pressure at the upstream and downstream position can be assumed to be uniform across the height of the tunnel. Assuming ideal flow near the wall, find the pressure difference P_A-P_B.

(iii) Again neglecting viscous force on the upper and lower walls, calculate the drag force per unit length on the model.

(iv) Estimate the total sideward force on the whole *full scale* submarine (for the conditions given at the beginning).

(e) In a separate model test, the model is placed in the middle of a deep open water tunnel. The velocity profiles in this case can be approximated as shown below. Note that while the height of the wake region is the same as that for the earlier model test *H=0.4m*, the velocity profile is now different not only inside the wake region but also outside the wake region:



Again ideal flow is assumed for flow away from the body and the pressure at the upstream and downstream position is assumed to be uniform across the height of the tunnel. If the force on the model is the same as that for the earlier model test (obtained in (d)(ii)), determine the value of α in this case.

[*Hint: Note that* α *<1 represents a loss of volume flux in the wake and must be accounted for by fluxes out from the top and bottom of the control volume. Follow the same steps you took in part (d) carrying* α *(and Q's expressed in terms of* α *) as unknown till the end. Equate the final result for the force you obtain with that in (d) to determine the value of* α *in this case.*]

(End of Quiz I)

Solution to Quiz 1

Section A (50%):

1 (6%): [$D\rho/Dt = 0$]

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\vec{v}) = 0$$

$$\nabla\cdot\vec{v} = 0$$

2 (4%): $\vec{v}\cdot\vec{\eta} = 0$

[fixed boundary]

3 (2%): $\frac{\partial \rho}{\partial t} + \vec{v}\cdot\nabla\rho = 0$

4 (6%): [incompressible] [linear relationship between viscous stress and rate of strain] [isotropic]

$\tau_{xx} = 4\mu x$

$\tau_{xy} = 0$

$F_x = 4\mu x \eta_x A$

5 (9%): $Re = 1.6\times10^{9}$ ($Re = \frac{UL}{\nu}$)

$Fr = 0.25$ ($Fr = \frac{U}{\sqrt{gL}}$)

$U = 100\ m/s$

$U = \sqrt{10}\ m/s$

$g = 1\times10^{4}\ m/s^2$

$\nu = \sqrt{10}\times10^{-8}\ m^2/s$

6. (3%): $2.8\times10^{4}\ N/m^2$ ($P_{bubble} = P_{atmosphere} + \rho g H + \frac{2\Sigma}{R}$)

7 (6%):
- (a) [true]
- (b) [true]
- (c) [false]
- (d) [true]

Section A. (contd)

8 (6%): Ideal fluid flow and conservative body force

$\omega_1 = \Gamma_1 / A_1$

$A_2 = 2A_1$

$\Gamma_2 = \Gamma_1$

$\omega_2 = \Gamma_1 / 2A_1$

9 (4%): $U_2 = 10\ m/s$

$P_2 = 50005\ N/m^2$

10 (4%): $A = 2 \quad (\nabla \times \vec{v} = 0)$

$B = 4 \quad (\nabla \cdot \vec{v} = 0)$

Section B (50%)

1.(20%):

(a) (i) Continuity equation: $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

(ii) N-S equation.

$$x. \quad v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}$$

$$y. \quad u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial y} + \frac{\mu}{\rho}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

(b) At $y=0$, $u=0$, $v=0$

At $y=H$: $u=0$, $v=0$

(c) $\frac{\partial u}{\partial x} = 0 \Rightarrow \frac{\partial v}{\partial y} = 0 \xrightarrow{v=0 \text{ at } y=0,H} v=0$ everywhere

(d)
$$v=0 \Rightarrow \left\{\begin{array}{l} \frac{\partial P}{\partial y} = 0 \Rightarrow P = P(x) \\ -\frac{1}{\rho}\frac{\partial P}{\partial x} + \frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2} = 0 \end{array}\right\} \Rightarrow \underset{\substack{\uparrow \\ \text{function of } x}}{\frac{dp}{dx}} = \mu \underset{\substack{\uparrow \\ \text{function of } y}}{\frac{d^2u}{dy^2}}$$

∴ $\frac{dp}{dx}$ is a constant

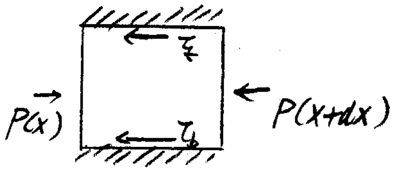
(e) (i) $u = \frac{1}{2}\frac{C}{\mu}(Hy - y^2)$

(ii) u reaches maximum value at $y = H/2$

(iii) $u_{max} = \frac{1}{8}\frac{C}{\mu}H^2$

(f) $\tau_t = \tau_b = \frac{1}{2}CH$, in positive x direction (stress on plate)

(g)



$$(\tau_t + \tau_b)\,dx = [P(x) - P(x+dx)]\,H$$

$$\Rightarrow \tau_t + \tau_b = CH$$

Symmetric flow $\Rightarrow \tau_t = \tau_b = \frac{1}{2}CH$

Section B (contd.)

2. (30%): (a) – (c): 10%, (d): 12%, (e): 8%

(a) ρ: ML^{-3}, μ: $ML^{-1}T^{-1}$, U: LT^{-1}

L: L, D: L, F: MLT^{-2}

(b) $\pi_1 = \frac{D}{L}$, $\pi_2 = \frac{F}{\rho U^2 L^2}$, $\Pi_3 = \frac{\rho U L}{\mu}$

(c) $L_m = 2m$, $U_m = 5\ m/s$

(d) (i) $-\rho U_m h + \rho \alpha U_m H + \rho \frac{7}{6} U_m (h-H) = 0 \Rightarrow \alpha = \frac{1}{6}$

(ii) $P_A + \frac{1}{2}\rho U_m^2 = P_B + \frac{1}{2}\rho (\frac{7}{6}U_m)^2 \Rightarrow P_A - P_B = \frac{13}{72}\rho U_m^2$

(iii) $-\rho U_m^2 h + \rho \alpha^2 U_m^2 H + \rho (\frac{7}{6})^2 U_m^2 (h-H) = (P_A - P_B)h + F_x$

$\Rightarrow \frac{F_x}{\rho U_m^2 H} = -\frac{1}{4} \Rightarrow F_x = -2500\ N/m$

$\Rightarrow$ Drag $= -F_x = 2500\ N/m$

(iv) Total force on the mode: Drag $\cdot L_m = 5000\ N$

Similitud of Π_2 and $\Pi_3 \Rightarrow F_s = F_M$

∴ Total force on the full-scale submarine: 5000 N

(e)

$$\begin{cases} -\rho U_m H + \rho \alpha U_m H + (Q_t + Q_b) = 0 \\ -\rho U_m^2 H + \rho \alpha^2 U_m^2 H + (Q_t + Q_b) U_m = (P_A - P_B) H + F_x \end{cases}$$

$$\left.\begin{array}{l} P_A = P_B \\ F_x = -2500\ N/m \text{ or } \frac{F_x}{\rho U_m^2 H} = -\frac{1}{4} \end{array}\right\} \Rightarrow \alpha = \frac{1}{2}$$